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FINAL REPORT

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Code - 1

RESEARCH on FUNDAMENTAL

ELECTRODYNAMIC and HYDRODYNAMIC

PHENOMENA INVOLVED

in

ELECTROMAGNETIC PUMPING

of

LIQUID METALS

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LIQUID METALS, INC.
WESTFORD, MASSACHUSETTS

Prepared by G. R. Findlay

SUMMARY

A theoretical analysis of the fundamental principles of magneto-hydrodynamic flow of liquid metals in any type electromagnetic pump is presented. The inherent advantages and potential limitations of each of four types of electromagnetic pump are discussed along with the major problems that affect design of a light weight, efficient and reliable pump. A study made of the various pumping requirements is reported. The potential areas of application for different type EM pumps in these space power plant systems is indicated. A detailed circuit analysis is made for the electrodynamic type pump. In addition, an example is presented of the weight and weight penalty possible for the electrodynamic pumps which might be used in a potentially typical 2,000 K(e) nuclear turbodynamic space power plant system.

To supplement theoretically calculated performances of a number of electrodynamic pumps, an exploratory experimental program is presented and discussed. This program includes performance measurements on six different types of electrodynamic pumps covering flow ranges to 65 gallons/minute with developed pressure to 250 psi and operating temperatures to 1400° F., and with sodium, NaK, and potassium as the working fluid. The problem areas recognized during the study as requiring more basic experimental data before an efficient, light weight, reliable pump can be designed are called to attention.

Useful data for system and component optimization studies are presented in graphic form.

INTRODUCTION

In the nuclear power plant systems contemplated for applications in space, it is anticipated that forced circulation of liquid metals at high temperatures will be necessary in one or more different circuits. The nature of the application to power plant systems is such that all components must operate with a high degree of reliability over long periods of time. Moreover the pumps, as well as other components, must be light weight and efficient so that minimum power is used. To meet these strigent requireme: ts, all types of pumps which could conceivably meet the requirements should be examined to determine which is best qualified for any specific application.

One class of pump having attractive characteristics is that class in which pumping action is produced by interaction of a magnetic field with a current flowing through the electrically conducting liquid metal; these are the "electromagnetic" or "magnetohydrodynamic" class of pumps. Attractive features inherent in this type of pump include (1) complete sealing of the liquid with no opportunity for liquid leakage at shaft seals or joints, (2) absence of parts moving in the liquid metal, (3) in certain types of this class of pump, absence of any moving parts, and (4) the ability to develop pressure without a high liquid velocity.

The first feature noted above is attractive from the reliability standpoint, particularly in the case of a system which must operate

3.

over a long period of time. Attainment of complete reliability with zero leakage over extended times for shaft seals operating in contact with high temperature liquid metals on one side and possibly exposed to high vacuum on the other side presents problems difficult to overcome. Knowledge of the erosion of metals moving at high relative velocity in high tempera`re liquid metals is at present incomplete, but evidence on hand indicates that metal erosion presents a potentially serious problem over long time operation; consequently, elimination of moving parts in the liquid metal has the potential for considerably increasing reliability.

In certain power plant systems such as thermionic converters or thermoelectric converters, which contain no moving parts, the capability of pumping liquid metal without introduction of moving parts is attractive, since it is generally recognized that introduction of moving parts contribute to unreliability.

The ability to develop pressure without high liquid velocities may be an extremely important advantage for the electromagnetic type pump, since liquid velocity affects two areas of operation. (1) In an electromagnetic pump, the liquid velocity can be limited so that erosion of the fluid-containing conduit can be minimized over the long operating life anticipated. In centrifugal mechanical pumps, on the other hand, there are areas where velocity of the liquid relative to the containment walls is high. If erosion in high temperature liquid metal systems is found to be highly velocity

4.

dependent (as is suggested by evidence from preliminary work),

the the ability to develop pressure without high liquid velocities

could be extremely important. (2) The second point of importance

re. liquid metal velocity is that with an electromagnetic pump

cavitation at the inlet to the pump can be controlled without a large

degree of liquid sub-cooling: i. e. , the liquid velocity at inlet can

be very low until the pump has built up pressure, so that cavitation

can be avoided without the necessity for sub-cooling the liquid much

below its condensation temperature. This feature of the electro-

magnetic pump is important since it affects the amount of radiator

area and hence weight required.

Against these potential advantages that the magnetohydro-

dynamic class of pumps seems to offer must be balanced the

general expectation that these pumps will be heavy relative to a

centrifugal mechanical pump and probably not as efficient. Part

of the purpose of this study was to investigate the fundamental

principles underlying operation of the electromagnetic class of

pump to determine any basic limitations which might make these

pumps either heavy for thier output or inefficient as compared to

mechanical pumps.

FUNDAMENTALS OF OPERATION OF

MAGNETOHYDRODYNAMIC PUMPS

As pointed out in the introduction, this class of pump is characterized by the fact that the pumping action is produced by interaction of (1) a magnetic field perpendicular to the direction of fluid flow with (2) an electric current flowing through the liquid metal perpendicular to both the field and the direction of the liquid metal flow.

The four different types of this class of pump selected for study have the following four elements in common: (1) The working fluid is an electrically conducting liquid metal which is incompressible and viscous. (2) The magnetic field is directed perpendicular to the direction of the liquid flow. (3) The electric current (designated in this report as a "conduction current") flows through the liquid metal perpendicular to the direction of the magnetic field and to the direction of the liquid flow. (4) A conduit contains the fluid being pumped (see Diagram 1).

The simplest structure representative of this type of pump is the so-called D. C. Faraday pump, in which a constant magnetic field is applied across a conduit containing the liquid metal through which a D. C. current from an external source is passed. The force developed in the liquid metal is proportional to the product of the magnetic field flux density x the interior dimension of the conduit in the direction in which current is passed x the current that is passed.

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LIQUID
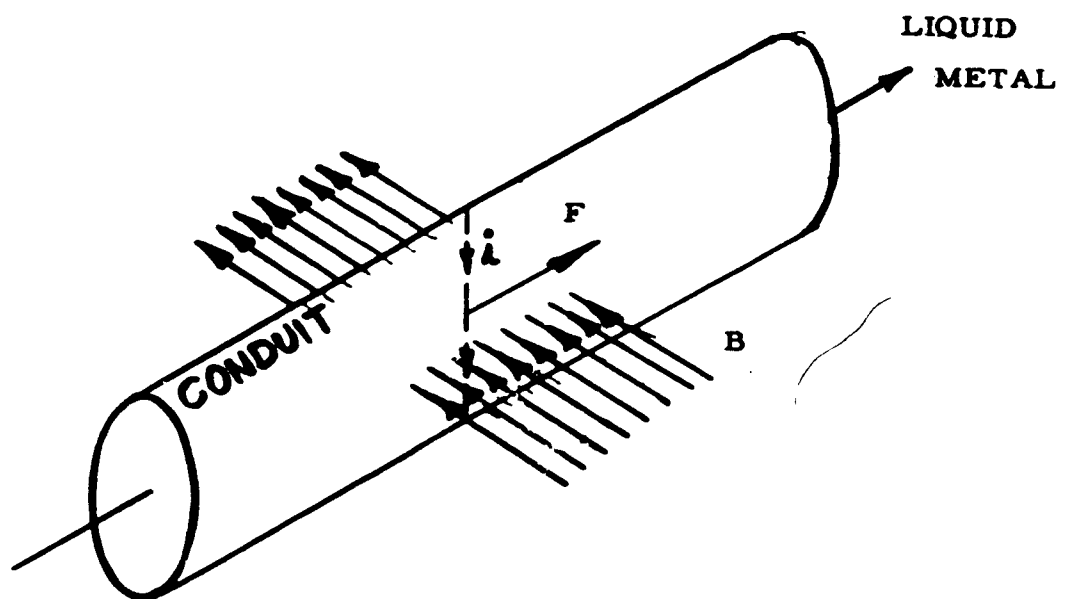METAL

F

i

B

CONDUIT

DIAGRAM 1

$$F = 0.57 \, B \, i \, \ell \times 10^{-6} \qquad \text{(Equation 1.)}$$

where:

F is the pumping force in pounds
B is the magnetic flux density in gauss
i is the electrical conduction current in amps
ℓ is the interior dimension in inches

With skillful design to minimize current losses that develop,

the D. C. Faraday pump may be an efficient pump, and it has no

moving parts. There is, however, one serious objection to this

type of pump, and that is the power supply required. The current

that must be passed through the liquid metal inherently is a current

of high amperage, low voltage. Power supplies producing thousands

of amps at one or less volts output are by nature inefficient and

heavy per unit of output power. In addition, the conductors leading

from the power supply to the oump must be large if high i^2 r losses

are to be avoided. The D. C. Faraday pump could, however, offer

attractive possibilities in nuclear power plant systems in which

conversion is accomplished by thermionic emission or thermo-

electric conversion, since both these converter units produce their

power as low voltage, high amperage D. C. Moreover, the feature of

no moving parts in this type of pump would go well with a thermionic

or thermoelectric conversion power plant syst..n, which similarly

has no moving parts. (It might be mentioned that one of the strongest

arguments against thermionic type converters is the problem of the

conversion equipment necessary to convert this high amperage, low

voltage D. C. current to a useable higher voltage A. C. power in an

8.

efficient, light weight, and reliable power conversion unit).

The second representative type pump of this class is the so-called A. C. conduction pump. The main advantage of this type pump is its successful circumversion of the problem of the high amperage power supply; the circumversion is accomplished by going to A. C. current by single turn transformer action. However, this conduction current, which flows through the liquid metal, is now an alternating current. In order for the pumping action to remain in the same direction, it is necessary to reverse the direction of the magnetic field at the same time that the direction of the current reverses. This is accomplished by applying an alternating magnetic field across the conduit to interact with the alternating conduction current. While this type of pump eliminates the power supply difficulty characteristic of the D. C. Faraday pump, the A. C. conduction pump retains all the other problems of the D. C. Faraday pump which will be discussed later in more detail. In addition it introduces new problems which affect its efficiency and its weight per unit output capabilities: (1) The wave form of the flux density in the liquid metal and the form of the current plotted against time are both approximately sinusoidal in shape. If the field and the current were exactly in phase, the pumping action would be entirely in one direction, but would have a pulsation equal to twice the frequency of the power supply, since twice each cycle both the flux and the current are zero, in which case the force also drops to zero.

In practice, however, a phase displacement between the induced conduction current and the magnetic field is present, with the result that over a portion of the cycle the direction of pumping actually reverses, and there is a pulsation in the output flow of liquid metal with a loss in efficiency. To help minimize the phase displacement, capacitors may be added to the circuit, but these considerably increase pump weight per unit output. (See Diagram 2)
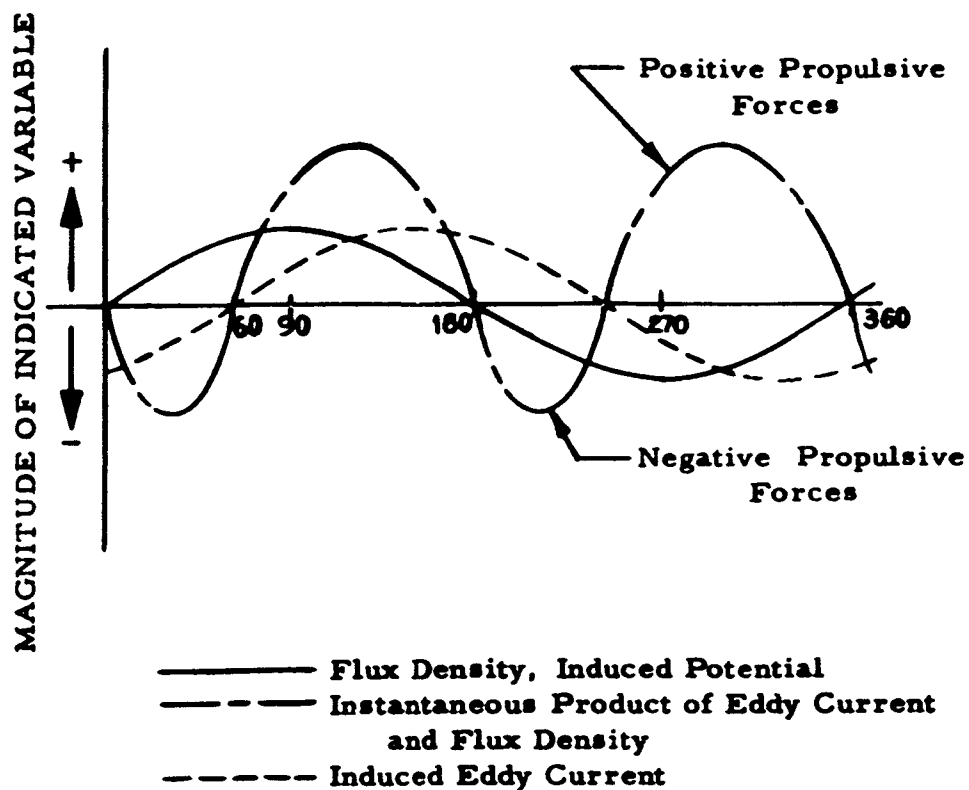
(2) The second complicating factor in producing a light weight A. C. conduction type pump concerns the efficiency of utilization of material in the magnetic circuit, by far the heaviest portion of these pumps. If a light weight pump is to be produced, a major design consideration is efficient utilization of the magnetic materials in the magnetic circuit. With an alternating magnetic field, the average useful magnetic flux density in the iron portion of the magnetic circuit is less than in the case of a continuous field, since for most of each cycle the magnetic flux density is less than the maximum; from its maximum positive value it decreases, passes through zero, and then builds up to a negative maximum. In addition, the alternating flux induces wasteful eddy currents and hystersis losses. As compared with the continuous flux magnetic circuit, the laminated A. C. magnetic circuit is clearly more efficient.

The third type of pump is the polyphase A. C. induction pump. In this pump some of the difficulties of the conduction type pump are eliminated; the high amperage alternating secondary current is

POLYPHASE MAGNETIC FIELD WITH EDDY CURRENT

LAGGING FLUX BY 60°

MAGNITUDE OF INDICATED VARIABLE

Positive Propulsive Forces

Negative Propulsive Forces

60 90 180 270 360

——————— Flux Density, Induced Potential
——— ——— Instantaneous Product of Eddy Current
            and Flux Density
— — — — — Induced Eddy Current

FOR NET PUMPING EFFECT, AREAS REPRESENTING
NEGATIVE FORCE MUST BE SUBTRACTED FROM THOSE
REPRESENTING POSITIVE PUMPING.

DIAGRAM 2

11.

induced in the liquid metal by having a number of polyphase A. C. electromagnetic poles arranged either linearly or helically along the liquid metal conduit. The efficiency of this type of pump is better than that of the A. C. conduction pump because it minimizes the so-called end losses where the current circumverses the magnetic field. But the polyphase A. C. induction pump, like the A. C. conduction pump, is penalized by the phase displacement between the sinusoidally varying magnetic flux and induced electric current in the liquid metal and by pump cell losses resulting from eddy currents induced in the pump cell wall and in the liquid metal by the sinusoidally varyingflux. In addition, as in the A. C. conduction pump, the alternating flux poses a problem regarding ability to utilize effeciently the magnetic materials in the magnetic circuit. Moreover, to obtain optimum performance with the polyphase A. C. induction pump, it is necessary that the frequency of the power supply be adjusted relative to the number of poles and to the liquid velocity. Usually, optimum effeciency lies at relatively low frequency, i. e. ; in the range of 10-20 cycles per second, and this creates a power conversion problem. Like D. C. Faraday pumps and A. C. conduction pumps, polyphase A. C. induction pumps do have the advantage of no moving parts.

The fourth type of pump considered is the electrodynamic pump. In this magnetic circuit produces a continuous field across the conduit, and in addition the entire magnetic circuit is mechanically

12.

moved to induce a current in the liquid metal contained in the conduit

As the magnetic poles of an electrodynamic pump pass over an element of fluid in the conduit, the rate of change, with time, of the magnetic flux through the element of fluid is a constant and then falls essentially to zero after the pole has slipped past the given element of liquid. Consequently, as seen from any element of the liquid, the wave form of flux density plotted against time is rectangular. Similarly, since the flux is constant during the period in which the pole is passing over the element of liquid, the induced potential in the liquid is constant with time, so that the product of current and flux density, which product produces the pumping force, is the vector product of two rectangles. The very slight phase displacement caused by the inductance of the secondary circuit displaces rectangular rather than sinusoidal forms, so that the loss in pumping is minimized. (See Diagram 3) Moreover, in the case of the electrodynamic pump, the field is slipping past the liquid and advancing in the direction of liquid flow more rapidly than the liquid is moving, so that the induced potential is in a direction to drive the current through the liquid so as to make the liquid attempt to follow the movement of the flux wave. In the case of a D. C. Faraday pump, on the other hand, the fluid is moving relative to the pole so that the induced potential of the conductor (i. e., liquid metal) moving through the magnetic field is a counter emf which tries to prevent the current from flowing through the liquid metal in the area of the magnetic field. The electrodynamic pump gains its potential advantage of

The liquid in the center of the conduit reacts as shown in the diagram as the pole faces approach, then pass over the element of liquid.

time during which opposed pole faces are moving over an element of fluid in the center of the conduit
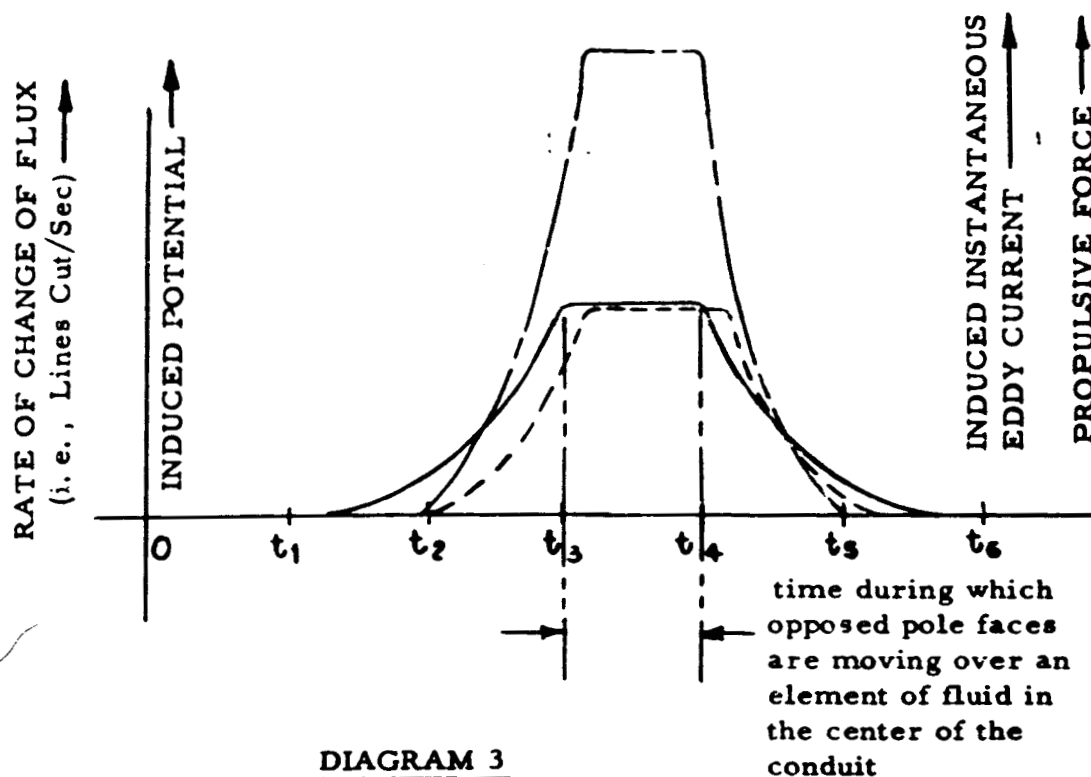
DIAGRAM 3

————— Flux Density, Induced Potential
—— — —— Propulsive Force (always +)
— — — — — Induced Eddy Current (instantaneous, always +)

Propulsive or pumping force varies as the product of instantaneous flux density and induced eddy current with square wave form of bodily moved DC magnetic field, eddy current lag does not seriously reduce pumping action. Note 1. With DC field, flux density and induced currents are always positive.

Note 2. With DC field, pumping action is always positive.

14.

lower weight and higher efficiency at the expense of having moving

parts, although the moving parts are not in the liquid metal. It

might best apply to turboelectric systems: such systems already

have rotating parts, and the addition of the pump rotor would only

slightly increase the existing degree of system unreliability. A

detailed circuit analysis and design calculation procedure are

presented in Appendix A for the electrodynamic pump.

In each of the four types of electromagnetic pump discussed

above, the material of construction of the pump cell or passageway

through which the liquid metal flows and the characteristics of the

liquid metal itself have a large bearing on the weight and efficiency

possibilities of the pump. In all cases it is desirable for the cell

construction material to have a high electrical resistivity in the

direction parallel to conduction current flow. At the same time,

the pump cell construction material must have adequate strength

and thickness to resist deformation by the internal pressure of

the li⁻ ⁻umped.

tic field configuration which makes maximum

use of the flux in the iron structure is that in which the gap is

small in comparison with the face dimensions of the magnet.

Hence from the magnetic circuit standpoint, the most desirable

shape for the liquid metal pump cell is a rectangle having its

dimensions in the direction of the conduction current flow large

in comparision with its thickness in the direction parallel to the

15.

flux.

This type od design requirement puts the large burden on the materials oi construction of the pump cell, since the span becomes long, and hence if the internal pressure is high the cell thickness must be great. At elevated temperatures the more common construction materials lose strength rapidly, so that very thick walls are required; increased wall currents and attendant losses result. In the case of the refractory metals, kept thinner and wall losses minimized. However, the refractory metals are in general much better electrical conductors than are the more common stainless steels and inconel. Fortunately, the stress problem is not an inherent problem; it can be handled by good design: i.e., by providing internal support for the cell so that the span between support points is not excessively great. Eddy current losses in the pump cell wall can be greatly reduced by other techniques, such as providing a very thin internal liner electrically insulated from a heavier walled external jacket. Such techniques complicate fabrication and can be justified only when weight and efficiency considerations are sufficiently important. Nevertheless, these techniques provide a solution both to the stress problem and to the wall loss problems.

On the other hand, the properties of the liquid metal cannot be changed once given a liquid metal is selected for a specific application. The heat capacity and density of the liquid metal

16.

determine the volumetric flow which must be handled by the pump to

transport the desired quantity of heat for a given temperature differ-

ence. Likewise the density and the viscosity of the liquid metal

at operating temperature determine the maximum liquid velocity

permissible before the hydraulic loss exceeds the optimum for

the system involved. The erosion and mass transfer characteristics

of the liquid metal in combination with the construction material

containing it may determine the maximum liquid velocity allowable

before the cell material erosion rate becomes excessive. Finally,

the electrical resistivity of the liquid metal determines the inherent

$i^2 r$ loss in the conduction currents flowing through the liquid metal

represents the major source of the inefficiency in the pump. For

pumps producing the same output work and operating at the same

efficiency but pumping different liquid metals, the size of the pump

is directly proportional to the electrical resistivity of the liquid

metal being pumped. The electrical resistivity of the alkali metals

increases considerably with increasing temperature; lithium

increases with temperature, but not so rapidly in the higher temp-

erature range (see NASA Report N62-15308). A few of the liquid

metals ahve electrical resistivities which actually descreas with

temperature, as in the case of zinc, or which remain almost

constant, as in the case of magnesium.

17.

PUMPING REQUIREMENTS IN NUCLEAR TURBOELECTRIC

SPACE POWER PLANT SYSTEMS

In order to gain some insight into the actual conditions under which pumps will be expected to perform in future power plant systems, information available about research programs either in operation or contemplated was reviewed; from this information, probable pump functions, liquid metals to be employed, and operating conditions were recoginzed as probable or possible. The pump function recognized as likely are as follows:

(1) Boiler Feed Pump

A boiler feed pump is a necessity in any Rankine cycle power plant system. The purpose of the boiler feed pump is to receive the liquid condensate from the condenser at condensation pressure and to increase its pressure not just to boiler pressure but sufficiently beyond boiler pressure to allow throttling before entrance to the boiler in order to stabilize boiling.

The boiler feed pump requires these general characteristics: (a) low to moderate volume in the range of tens to hundreds of gallons per minute for the power plants likely to be considered in the future; (b) high developed pressure: i.e., from hundreds of pounds per square inch to as much as a thousand pounds per square inch; (c) high total syatem pressure: i.e., to 1000 pounds

18.

per square inch; (d) moderate operating temperature: i.e., in the 1400 F. range, or lower in the case of some systems like SNAP 8; (e) the materials of construction at the present time are difficult to determine but most likely will be refractory metals required for the high temperature portion of the system; (f) the fluid will be determined by the maximum allowable system temperature limits imposed by the materials of construction: i.e., for the lower range of operating temperatures (maximum, 2000° F.) rubidium or potassium would likely be the fluid, while for the higher temperature range (e.g., 2500° F.) sodium would probably be the working fluid; (g) special provisions will be necessary at the pump inlet to protect against cavitation when operating with the liquid entering at very low net suction inlet head. In addition, special consideration will have to be given to the case of separating the vapor from the liquid at inlet when operating at a zero field where cavitation is likely.

(2) Primary Heat Exchange Pump

In most nuclear turboelectric power plant systems this type of pump would likely be required to pump the coolant which transfers heat from the nuclear source (i.e., the nuclear reactor field elements) to the boiler, where the heat is then transferred to the working fluid. The same heat transfer fluid would probably be used to superheat the vapor working fluid in a separate

superheater, so that the vapor would be dry when entering the turbine. In addition, the same heat transfer system would probably also be used for at least one stage of reheat of the vapor at some intermediate pressure between the boiler and the condenser pressure. This requirement would probably be necessary from the standpoint of keeping the expanding vapor dry to prevent excessive erosion.

The primary heat exchange pump would have the following characteristics: (a) high volume throughput, i. e. , from hundreds to thousands of gallons per minute for the power plants under consideration, (b) relatively low developed head, probably less than a hundred pounds per square inch; (c) moderately high total system pressure, probably less than 200 pounds per square inch; (d) high operating temperature, 2000 F. or above; (e) refractory metal construction; (f) lithium, sodium, or possibly magnesium as the circulating fluid; (g) to eliminate problems arising from cavitation and from the effects of zero g environment, the system would be a pressurized one.

(3) Pump to Circulate the Heat Exchange Agent for Removal of the Heat of Condensation from the Condenser and for Transfer of This Heat to the Radiator for Radiation to Space

This pump would have most of the same characteristics as the primary heat exchange pump in turboelectric power plant systems with the exception that its in turboelectrci power

plant systems with the exception that its operating temperature would be much lower, probably in the $1400^\circ - 1500^\circ F$. range. Again, its capacity would be high, in the hundreds to thousands of gallons per minute; it would develop a moderate pressure; and the heat exchange agent would likely be sodium in advanced systems and NaK in early systems. The materials of construction of the pump cell would be determined by the construction materials of the radiator and of the condenser. Again a pressurized system would be used to prevent problems presented by cavitation and the effects of zero g.

While the above three pumps would account for the major liquid circulation requirements in most turboelectric power plant systems for space applications, several other pumps of a secondary service or importance may be required:

(1) Coolant Start-Up Pump

For example in SNAP 8 a battery-operated D.C. Faraday pump is used to give 10% of flow capacity for the four hour period when the reactor is being started and before generator power is available to operate the main circulation and boiler feed pumps.

(2) Pump to Circulate the Heat Transfer Liquid Metal for Turbine Interstage Reheat

A separate pump may be desirable for this function beacuse of the larger pressure drop that may be required in the heat

21.

exchange passageways of the reheat radiator. This pump

would pick up a portion of the primary heat exchange agent

stream as it leaves the reactor and circulate the liquid through

the interstage reheat units. An important characteristic of the

pumping requirements in these systems is that the system

operate at constant capacity so that pump outputs do not have to

vary, but run at their full nominal capacity at all times. This

eleminates much complication, and in the case of electrodynamic

pumps allows construction of a pump having having no require-

ments for electrical power, but only for a mechanical drive

for the rotor: the magnetic field of the electrodynamic pump

might in this case be produced by permanent magnets rather

than by electromagnets.

To determine the effect which selection of a specific liquid

metal has on performance of an electromagnetic pump in a turbo-

electric power plant system, the case of the boiler feed pump was

considered when the working fluid was changed from soduim to

potassium to rubidium to cesium to mercury. The first step was to

determine for power plants having the same output the relative number

of pounds of each of these different fluids which have to be circulated

per unit time in order to obtain the same power outputs. Ideally

this information would be obtained by plotting an actual cycle on a

Mollier diagram for the fluid in question and then determining the

theoretical cycle efficiency and the actual power output per pound

of fluid circulated per unit time. However, accurate Mollier diagrams constructed from actual experimental data were not available. The calculated Mollier diagrams for sodium and potassium were not believed to offer sufficient accuracy to plot an accurate cycle.

To make this comparative study, the following course was taken: the heat of vaporization per pound for each fluid was taken as a representative value after graphs had been plotted (Drawing C-11404) to show the relative amount of heat per unit power output that would be required for each of the separate working fluids, the available information on liquid heat content, heat of vaporization, and the vapor heat content for the liquid metals under consideration. The value of heat of vaporization per pound for each of the liquid metals is tabulated in Table I, line 1. Since the theoretical mechanical work required for the boiler feed pump is a product of the volumetric flow per unit time and the pressure which must be developed, the vaporization heat requirement has to be expressed in B. T. U. per cubic foot of liquid pumped. This figure for each of the liquid metals shown in line 3, was obtained by using the data of line 1, along with the density of the liquid as obtained from Drawing C-11405, where it had been previously plotted. In the case of alkali metals, the density was taken at a temperature of $1500\,^{\circ}F$. while in the case of mercury the density was taken at about $500\,^{\circ}F$., which would correspond to its condensation temperature. The densities of the liquid metals used are tabulated in line 2.

The next assumption made was that the different liquid metals would all have to be pumped through the same pressure differential. This assumption is reasonable because the liquid metal to be selected as the working fluid will be determined by the construction materials and the maximum allowable operating temperature imposed by these materials. Once a constant developed pressure for all potential working fluids is assumed, the pumping work then becomes proportional to the volume of liquid which must be pumped. As a basis for this study, sodium was chosen as having a theoretical pumping work value of 1; the theoretical pumping work for each of the other liquid metals is given by dividing the B. T. U. per cubic foot for sodium by the equivalent B. T. U. per cubic foot for the particular liquid metal under consideration. The relative values of required work are tabulated in Table I, line 4. The significance of these figures is that they show, for example, that if potassium rather than sodium were the working fluid, and if the boiler feed pump is assumed to have 100% efficiency, the work required for potassium would be 2.4 times the work required for sodium. Similarly, in the case of mercury the work required would only be 0.856 times the work required for sodium, because the sodium has a lower liquid density relative to its heat of vaporization than has mercury.

However, in addition to the inherent pumping requirement and

the assumed 100% efficiency, other factors must be considered: e.g., the relative ability of any liquid metal to be pumped determines the size of the pump required to operate with that liquid metal at a given efficiency. For example, analysis of the basic equations for electrodynamic pumps shows that for pumps of equal efficiencies, the size and hence weight of the pumps are inversely proportional to the liquid velocity in the pump cell. The liquid velocity in the pump cell will be limited by one of two conditions: (1) the erosion limit which is acceptable over the desired operating life of the pump (unfortunately no data are available to enable comparative ratings of the various liquid metals under consideration as to maximum allowable velocity if erosion proves to be the limiting factor); (2) If erosion and mass transfer do not impose limitations, then the limiting factor will be the excessive hydraulic loss which would eventually result as the liquid velocity is increased. The fluid velocities through the pump cell that will produce equal hydraulic losses for each of the various liquid metals are inversely proportional to the square root of the liquid metal; this assumes turbulent flow, which would always be present in the systems under consideration, and it disregards the small effect of change in density on the change of friction factor with Reynolds number. Again taking the case of sodium having a value of 1, the comparative pump size for equal hydraulic efficiencies resulting from the allowable velocity is calculated and tabulated in line 6. This shows, for example, that

25.

to have the same volumetric capacity and developed head as a pump for sodium, a pump for potassium would be only 0.946 times as big as the pump for sodium. The potassium pump would be smaller because being less dense the potassium can have a slightly higher liquid velocity than can sodium for the same hydraulic loss. Conversely, because of the velocity factor, the pump for mercury, operating under equal volume conditions and equal hydraulic efficiency conditions, would have to be 4.5 times as big as a sodium pump.

The third significant factor that affects the relative size of pumps for different liquid metals is the electrical resistivity of the fluid. As mentioned previously, the inherent $I^2 R$ loss of the conduction current passing through the liquid metal is proportional to the electrical resistivity of the liquid metal being pumped. Consequently, in pumps having equal capacities, for equal $I^2 R$ losses to be obtained the cross sectional area for the conduction current must be proportional to the resistivity of the liquid metal being pumped. In line 7, Table I, the electrical resistivities of the liquid metals under consideration are tabulated in units of micro-ohms-centimeters. These values are taken at the temperatures previously indicated for the liquid metals considered. In line 8, the ratios of the resistivity of each of the liquid metals to the resistivity of sodium, which is taken as 1, are compared. This table shows that for pumps having equal output and operating with

equal efficieny, in order for the inherent I^2R conduction loss to be the same, the pump for potassium for example would have to be 1.7 times the size of the pump for sodium, and the pump for mercury would have to be 2.47 times the size of the pump for sodium in terms of weight.

Now, each of these individual factors, i.e., liquid velocity, volumetric flow for a given power plant output per unit time, and electrical resistivity, must be multiplied together to give the relative weight of the pump that operates under equivalent efficiency conditions as compared to sodium. The product of these three factors is tabulated in line 9, and shows for example that under the same circumstances the pump for potassium would weigh 3.86 times as much as the pump for sodium. The pump for cesium would weigh 19.3 times as much as the pump for sodium, and the pump for mercury would weigh 9.5 times as much as the pump for sodium. As has been pointed out in this analysis, many assumptions have been made that are not quite rigorously proved in practice. Nevertheless, analysis does show the relative difficulty of pumping these liquid metals to feed the boiler.

This same kind of analysis may be carried out for the primary heat exchange pump, with the difference that instead of the heat of vaporization per cubic foor of liquid, the quantity to be considered would be the heat capacity per unit volume of fluid. The liquid velocity correction factor and the electrical resistivity correction

27.

factor would remain the same. This analysis was carried out for lithium, sodium, and magnesium at 2000° F. From this analysis lithium would offer some advantages as the high temperature primary heat exchange fluid. However, another important point which must be kept in mind is that the analysis is based on the assumption that the liquid velocity would not be limited by erosion or mass transfer. It is conceivable that when the limiting liquid velocity is determined by liquid density the fluid which looks most attractive from the above analysis might not in the end be the best choice: e.g., if the liquid itself were more aggressive in its attack on construction materials, so that in practice the liquid velocity might have to be reduced more than might be the case for one of the other liquids which in the initial evaluation may not have looked quite so promising. For example, lithium is known to be quite aggressive in its attack on most of the construction materials. The choice of containment at present would seem to be limited to columbium-base alloys. On the other hand, the small and admittedly inadequate corrosion information available on magnesium would indicate that its attack on construction materials might not be so severe as that of lithium, and that a wider range of refractory metal alloys might be possible as construction materials if magnesium were the heat exchange fluid. This is mentioned only to point out clearly the importance of getting actual operating experimental data on erosion and mass transfer for potential liquid metals and

28.

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their associated construction materials.

PUMP SIZE PARAMETERS

1. Heat of vaporization BTU per pound	1,800	875	340	225	117
2. Density lbs per cu ft (Hg 500 F; others 1500 F)	47.2	40.5	30	101	848
3. BTU per cu ft of liquid pumped	85,000	35,000	27,200	22,700	99,200
4. Required work relative to Na	1.0	2.4	3.12	3.75	0.856
5. $\dfrac{1}{\sqrt{\text{Density}}}$	0.115	0.163	0.128	0.10	0.0346
6. Velocity of liquid metal relative to Na	1.0	0.946	1.206	1.545	4.5
7. Electrical resistivity micro-ohm cm (Hg 500 F; others 1500 F)	48.5	82.5	125.0	162.0	120.0
8. Resistivity relative to Na	1.0	1.7	2.59	3.34	2.47
9. Relative pump weight 4 x 6 x 8	1.0	3.86	9.7	19.3	9.5

TABLE I

Experimental Test Program:

During the course of the present study program a limited experimental investigation was undertaken in order to determine how closely actual performance matched calculated theoretical performance, for electrodynamic type pumps. This limited experimental program also had a secondary objective, which was to gain information as to how the test loop should be constructed and instrumented to give the type of data that would have significance.

In all, seven different electrodynamic pumps were tested during the program. Data are presented in Appendix III. These pumps had been constructed for various experimental applications and ordinarly would have been tested only to determine the operating point. In the series of experimental runs that were made in this program more extensive instrumentation was supplied to the loop, so that inform- ation about the power input and power losses could be obtained. In all cases the loop was a simple circular or rectangular loop contain- ing a pressure gauge at discharge, a throttling valve on the outlet side, an expansion tank at the top of the loop, and a pressure gauge or pressure transmitter at the suction inlet side of the pump. No provision was made in these loops for a cooler to maintain the temperature constant. However, in most cases trace heaters were prcvided to put additional heat into the loop. Lack of provision for adequate cooling was one of the main limitations inherent to these tests since it was difficult to maintain a constant loop temperature

30.

during the series of operating runs, particularly at low temperatures. This was particularly difficult and troublesome in the case of the pumps having columbium pump cells where the temperature had to be held below 500° F. to prevent damage to the columbium since it was not completely jacketed.

In most cases pressure was measured through an oil-filled transmitter to a regular Bourdon pressure gauge. This system worked adequately in the case of potassium, but considerable difficulty was encountered with oxide plugging in the case of sodium. In the case of tests using NaK, the NaK was allowed to enter the Boudon gauge directly. All-welded stainless steel Bourdon tubes were used. The ever-present danger of getting oil into the loop makes the oil transmitter type pressure system somewhat undesirable and definitely not recommended for long time test operations. For the future consideration is being given to balanced pressure inert gas pressure diaphragm type gauges operating at loop temperatures to prevent cold trapping and plugging. In all cases flow was measured by an electromagnetic flowmeter whose performance was calculated by measuring accurately the field strength of the magnets and the dimensions of the tube. It is felt that in future test operations it would be desirable to have a second independent electromagnetic flowmeter having a different field strength and a different liquid velocity for comparison purposes.

The motor speed and the pump speed were determined by a

31.

General Radio Stroboscope. While the Stroboscope was very adequate for determining the speed, it was not too convenient since the data had to be taken each time a change was made in operating conditions. In all of these tests the speed of the pump was essentially fixed unless the pulleys were changed. From the standpoint of getting data to evaluate theoretically calculated performances it would be much more desirable to have a completely variable pump speed range which could be easily changed so that at a fixed pump field setting (fixed flux density) the speed could be changed to obtain a series of performance points.

In later runs the power input into the pump motor was determined by use of a polyphase watt meter with potential transformers to reduce the voltage from 440 to 110, and with current transformers to give the proper input for the Genreal Electric polyphase watt meter employed. This method was adequate for measuring the power into the motor, but it was still necessary to calculate the net power input into the pump from the motor efficiency curve. In the future it would be desirable to have a torque dynamometer on the pump shaft so that the actual pump torque can be measured as well as the pump speed. However, in spite of the limitations imposed by the lack of extensive instrumentation, a considerable amount of data covering a fairly wide a range of operating conditions was obtained which gave a fairly good quantitative view of theoretical performance compared with measured performance.

The largest cause of deviation between theoretical and measured performance appears to be in the bonding between the silver conductor and the stainless steel pump cell causing a high resistance contact which adds to the series resistance of the eddy current. The effect of this bonding was clearly seen in the case of the pump 17012/2. The first time the conductor was cast, the bonding was poor and the performance of the pump was considerably under design. It was necessary to raise the pump rpm to 1300 from 920 in order to meet performance, which was then only marginal. As a result the silver was melted out of this tube and recast, taking added precautions to insure a high conductivity junction. After the second casting the pump performance, even at 920 rpm, far exceeded the required design and was comparable to what would be expected from the theoretical performance curve when account was also taken of the effect of fringing flux contributing to the eddy current.

The second factor which seems to make a difference in the performance of actual pumps compared to the theorectically calculated performance is the effect of the electrical resistivity of the liquid metal and the construction material. In many cases the electrical resistivity of either the fluid or the construction material was not adequately known at the operating temperature used, and errors in the estimates that were made could have a considerable effect on the shape of the performance curve.

33.

APPENDIX   A

The following section presents a theoretical analysis of the para-
meters to be considered in designing an electrodynamic type pump
for circulating liquid metals.   The presentation is divided into three
sub-sections as follows:

I.   Hydraulic losses through the pump cell at design operating
        conditions.

II.   Electrical parameters of the eddy current circuit through
        the pump cell under nominal operating conditions for two
        cases: vis., (2.1) where the inner and outer conductor rings
        contain a material different from the liquid metal conduit
        material and where the magnetic poles on one side of the cell
        are of the same polarity, and (2.2) where the inner and outer
        conductors are of the same material as that containing the
        liquid metal and where on one side of the pump cell the magnetic
        poles are of alternate polarity.

III.   Calculation of the theoretical performance curve for an
        electrodynamic pump.

        A listing of all symbols used, together with their
respective definitions and units, is given; these symbols are further
clarified by reference to the ten figures also included in this section.

SYMBOL		DESCRIPTION	UNIT
A	=	Cross sectional area through which eddy current flows	sq. in.
a	=	Semi major axis of ellipse	inch
B	=	Magnetic flux density across the gap between magnetic poles	gauss
b	=	Semi minor ellipse axis	inch
C	=	Pump capacity or flow rate	gpm
C=O:	=	Blank - off condition	gpm
C=max:	=	Free flow (i.e., zero pressure condition)	gpm
c	=	1/2 the magnetic pole face length in the circumferential direction	inches
c_1	=	constant in the pressure equation, and is 4.32 x 10	
c_2	=	constant in formula for perimeter of 2:1 ellipse	dimensionless
(ci)	=	"inner conductor" - used as a subscript	
circ.	=	"circumferential" - used as a subscript	
(co)	=	" outer conductor" - used as a subscript	
D_t	=	Hydraulic diameter	ft.
E_L	=	Electrical potential generated in the liquid metal by rotation of the magnetic pole pairs	volts
E_{w_1}	=	Electrical potential generated in the liquid - metal-containing tube walls bridging across the electrical conductors and under the magnetic poles	volts
F	=	Force required to be generated per average eddy current in order to meet design specifications	lbs.

SYMBOL		DESCRIPTION	UNIT
F_c	=	Frictional loss of mechanical energy at entrance contraction of elliptical pump cell when liquid metal passes from round to elliptical section	ft.
F_e	=	Frictional loss of mechanical energy at exit expansion of elliptical pump cell when liquid metal passes from elliptical to round section	ft.
f	=	Friction factor	dimensionless
G	=	Mass flow	#/sq. ft. -sec.
g_c	=	gravitational constant, 32.2	ft/sec
h_{ci}	=	thickness of inner high conductivity element of pump cell	inches
h_{co}	=	thickness of outer high conductivity element of pump cell	inches
i_1	=	electrical eddy current flowing through liquid-metal containing tube walls bridging between the electrical conductors and under the magnetic poles	amps
i_2	=	electrical eddy current flowing through the liquid metal in the area between an opposed pair of magnetic poles (i. e. , directly under a magnetic pole face)	amps
$(i_1 + i_2)$	=	total electric current flowing in one average eddy current return path: i. e. , through the electrical conductors, and, if the magnetic poles on one side of the pump cell are all of the same polarity, through the liquid metal and containing tube walls in the average return eddy path outside the magnetic flux field	amps
K_c	=	constant = 0.05	
L_e	=	general term for any equivalent length	ft.
L_1	=	equivalent length of large circular portion of cell	ft.

SYMBOL		DESCRIPTION	UNIT
L_2	=	equivalent length of the two smaller radius curved sections at entrance and exit portions of large circular portion of cell	ft.
L_Σ	=	Toal equivalent length of curved portions of pump cell	ft.
(LM)	=	"liquid metal" - used as a subscript	
ℓ	=	length of path along which one average eddy current flows	inches
m	=	dimension of magnetic pole face in radial direction	inches
n	=	number of magnetic pole pairs	dimensionless
n_{ci}	=	circumferential length of average eddy current path through inner cinductor ring	inches
n_{co}	=	circumferential length of average eddy current path through outer conductor ring	inches
nom	=	"nominal" (i.e., design) conditions. Used as a subscript	
ΔP_{circ}	=	total pressure drop in liquid metal around all curved portions of liquid-metal-containing passage in the pump cell	psi
$\Delta P_{entrance}$	=	pressure drop at entrance contraction in ellipsed pump cell when liquid metal passes from circular to ellipsed section in straight portion at cell inlet end	psi
ΔP_{exit}	=	pressure drop through sudden enlargement in ellipsed pump cell when liquid metal passes from ellipsed to circular section in straight portion at cell discharge end	psi
q	=	point in return path of electric eddy current	
q_1	=	point in return path where eddy current from central tube enters high conductivity section of inner conductor ring	

SYMBOL	DESCRIPTION	UNIT
q_2	= mid point of radial return path between adjacent magnetic poles (this area has minimum magnetic flux)	
q_3	= point in return path where eddy current from high conductivity section of outer conductor ring enters central tube	
R	= Resistance, at operating temperature	ohms
R_{w_2}	= combined electrical resistance of both liquid-metal-containing tube walls (in parallel) bridging the two electrical conductor elements of the cell in the return path of one average eddy current through the space not covered by magnetic poles	ohms
r	= radius or radial	inches
r_1	= radius from center of pump cell to average eddy path in circumferential direction, or through inner conductor ring	inches
r_o	= radius from center of pump cell to average eddy path in circumferential direction, or through outer conductor ring	inches
r_h	= hydraulic radius	ft.
R_e	= Reynolds Number	dimensionless
S	= distance separating nearest edges of adjacent magnet pole faces in circumferential direction along center line of central tube of pump cell	inches
V_1	= velocity of liquid metal in ellipsed section of pump cell	ft/second
V_L	= velocity of liquid metal, general symbol	ft/second
V_2	= velocity of liquid metal in round section of pump cell	ft/second
V_R	= rotor velocity	ft/second

SYMBOL	DESCRIPTION	UNIT
V_S	= slip velocity	ft/second
$(W_{co})_1$	= radial length of average eddy path through outer high conductivity element	inches
$(W_{ci})_1$	= radial length of average eddy path through inner high conductivity element	inches
X	= ratio of centerline curvature (diameter) of tube to hydraulic diameter of tube interior	dimensionless
y_t	= ht. of ellipse ordinate at point where $X = 0.85$ a	inches
R_{ci}	= Total electrical resistance in one average eddy current path through the inner conductor member	ohms
$R_{(ci)_{circ}}$	= circumferential component of electrical resistance in one average eddy current path through the inner conductor member	ohms
$R_{(ci)_r}$	= radial components of electrical resistance in one average eddy current path through the inner conductor member	ohms
R_{co}	= total electrical resistance in one average eddy current path through the outer conductor ring	ohms
$R_{(co)_{circ}}$	= circumferential components of electrical resistance in the average eddy current path through the outer conductor member	ohms
$R_{(co)_r}$	= radial components of electrical resistance in an average eddy current path through the outer conductor member	ohms
R_{L_1}	= electrical resistance through the liquid metal under the magnetic poles in one average eddy current path	ohms

SYMBOL	DESCRIPTION	UNIT

$R_{(L_1 + \perp walls)}$ = Electrical resistance through the liquid metal and through the portion of the tube walls at each end of one average eddy current path adjacent to the conductor bars and under the magnetic poles ..... ohms

$R_{(\perp walls\ under\ magnetic\ poles)}$ = electrical resistance through the thickness of the tube wall separating the liquid metal from the high conductivity material of the inner and outer conductor rings (or bars) on both ends of the average eddy current path through the liquid metal under the magnetic poles ..... ohms

R_{w_1} = combined resistance of both (in parallel) liquid metal containing tube walls bridging between the two electrical conductor elements (or bars) of the cell and under the magnetic poles ..... ohms

z = wall thickness of liquid-metal-containing tube of pump cell ..... inches

α = subtended arc of circular portion of cell ..... degrees

α_1 = angular measure of largest arc of pump cell ..... degrees

α_2 = angular measure of smaller arcs at entrance and exit of pump cell ..... degrees

δ = (1/2) angle between magnetic pole face centers ..... inches

ρ = density of liquid metal ..... #/cubic feet

σ = resistivity, at opening temperature, of specific material through which eddy current flows ..... ohms-inches

σ_{LM} = resistivity, at operating temperature, of liquid metal being pumped

SYMBOL	DESCRIPTION	UNIT
μ	= viscosity of liquid metal (at operating temperature)	#/ft. second

L HYDRAULIC LOSSES (All numerical values substituted in utilizing the following design formulae should be those prevailing at nominal operating conditions (i.e., design conditions) of pressure, temperature, and flow rate.)

1.1 Through Curved Portions of Pump Cell

$\Delta P_{circ.}$ = Pressure drop around curved portions of liquid metal-containing passage.

$$= \frac{c_1 \, f \, (L_\Sigma) \, G^2}{D_t} \text{ psi}$$

1.1.1

c_1 = constant = 4.32×10^{-4}

1.1.2

f = friction factor, obtained from graph[1] of f vs Reynolds Number, where Reynolds No. =

$$Re = \frac{D_t \, G}{\mu}$$

and here
$$D_t = 4 \, r_h = \frac{\text{internal area of cross section of liquid metal-containing passage, sq. ft.}}{\text{wetted perimeter of LM-contg. passage, ft.}}$$

where
(1) r_h is the hydraulic radius, ft.

(2) the wetted perimeter factor (i.e., the denominator), ft.

for a circular passage = $\frac{\pi \, (L.D.'')}{12}$ ft.

for a 2:1 elliptical passage = $\pi 3 b'' c_2 / 12$, ft.

where
b = half minor axis of ellipse, and is determined by setting ellipse perimeter equal to the perimeter of the round tube used to form the ellipse:

$$\pi \, L.D.'' = \pi 3 b'' c_2$$

c_2 = constant = 1.028

Note [1]: Chem Eng'g Handbook
3rd Ed. Perry
McGraw Hill (1950) p. 382

42.

(3) the internal area factor (i.e., the numerator)

for a circular tube $= \dfrac{(\text{circular tube I.D. ''})^2 \, \pi}{4 \times 144}$ square feet

for a 2:1 elliptical tube $= 2\pi b^2$ (see detn. b above)$/ \, 144$ sqft

G = mass flow = $(V_L)(\rho_L)$ . . . .

    in which

       V_L = velocity of liquid metal, fps

       ρ = density of liquid metal, #/ cu. ft.

μ = viscosity of liquid metal (at operating temperature) = (centipoises / 1488), lbs./ft. sec.

1.1.3

$$L_\Sigma = L_1 + L_2$$

where

    L_Σ = total equivalent length of curved portions of pump cell, ft.

    L_1 = equivalent length of large circular portion of cell, ft.

    L_2 = equivalent length of the two smaller-radius curved sections at entrance and exit portions of large circular portion of cell, ft.

and

    $L_1 = L_e D_t = (0.0202 \propto^{1.1} X \; Re) D_t$

where

    $\propto$ = arc of large circular portion of cell, for which equivalent length, L_1, is required; degrees X is obtained from graph (*2) of X vs. D_c / D_t, and

    D_c = 2 x radius of curvature of bend in tube or pipe, in ft.

    D_t = as under section 1.1.2

  $L_2 = 2 \, (0.0202 \propto_2^{1.1} X \; Re) D_t$

where

    X, Re, D_t are as for L_1 above

    $\propto_2$ = arc of small-radius bend at entrance (and exit) portion of cell (see Fig. 1.), degrees

Note *2 : Chem. Eng'g Handbook, 3rd Ed. Perry. McGraw Hill (1950) p. 390

1.2 **Through Straight Exit Portion of Tube Cell, if Curved Portions of Cell are Ellipsed and Straight Exit Length of Tubing is of Circular Section**

ΔP_{exit} = pressure drop through sudden enlargement,

$$= F_e \frac{\rho}{144} \text{ psi}$$

where

$$F_e = \frac{(V_1 - V_2)^2}{2g}$$

in which

V_1 = liquid velocity in ellipsed section, f. p. s., and is same as calculated for G under 1.1.2 above

V_2 = liquid metal velocity in round section of tube

$$= \frac{\text{#}}{\text{min. x 60}} \times \frac{1}{\rho} \times \frac{1}{\text{cr. sec. area}} \text{ , f. p. s.}$$

$$= V_1 \text{ (internal area ellipse/internal area circ. tube)} \quad \text{(see Sec 1.1.2 (3))}$$

1.3 **Through Straight Entrance Portion of Tube Cell**

$$\Delta P_{entrance} = F_c \times \frac{\rho}{144}, \text{ p. s. i.}$$

where

$$F_c = K_c \frac{(V_{downstream})^2}{2 g_c} = K_c (V_1^2 / 2 g_c)$$

$$K_c = 0.05 \ldots \ldots \ldots \text{ *3}$$

$V_{downstream} = V_1$ of section 1.2 above

$g_c = 32.2 \text{ ft/sec}^2$

1.4 **Total Hydraulic Loss through Pump Cell at Nominal Conditions**

$\Delta P_{circ.}$ from Section 1.1 , + ΔP_{exit}, from Section 1.2 , + $\Delta P_{entrance}$, from Section 1.3 ; p. s. i.

Note *3 : See Heat Transmission, 2nd Ed. Wm. H. McAdams. McGraw Hill Book Co., Inc. 1942, p. 122.

II. CALCULATION OF ELECTRICAL PARAMETERS INVOLVED IN FLOW OF EDDY CURRENTS IN PUMP CELL

2.1   If the pump cell has, attached to the liquid metal-containing section of the cell, high-electrical-conductivity pieces of or containing a material different from the material housing the liquid metal in the cell, then

2.1.1.   Relative Physical Dimensions of Cell, and Associated Magnetic Poles, Required for Electrical (Eddy) Circuit Calculations:

2.1.1.1

In Fig. 1, let

(1) Magnetic poles on one side of the tube be all of same polarity; on the other side of tube, of opposite polarity.

(2) Number of magnetic pole pairs = n

(3) Average eddy paths be as indicated by arrows: number of such paths = 2n

2.1.1.2

Figure 2 shows Section AA' BB' of Figure 1, exploded, and the average path of one of the 2n eddy currents indicated with dimensions shown in substance.

In Fig. 2,

$$m = (0.90)(0.85)(2a)$$

where

2a = internal major axis of ellipse if tube section is elliptical, inches

= I.D. if tube is circular in section

n_{co} = (1/2 separation between adjacent magnetic poles on one side of cell + 1/4 magnetic pole face major dimension)$(r_o/r_{cell\ ₵})$, inches

= $(s/2 = c/2) (r_o/r_{cell\ ₵})$

n_{ci} = $(s/2 = c/2) (r_i /r_{cell\ ₵})$, inches

δ = 1/2 angle between magnetic pole face centers, degrees

$s/2$ = 1/2 distance separating facing edges of two adjacent magnetic poles of same polarity as measured along the centerline of the LM-containing tube, inches

45.

NOTE: FLUX NORMAL TO
PLANE OF PAPER AND
COMING OUT FROM PAGE
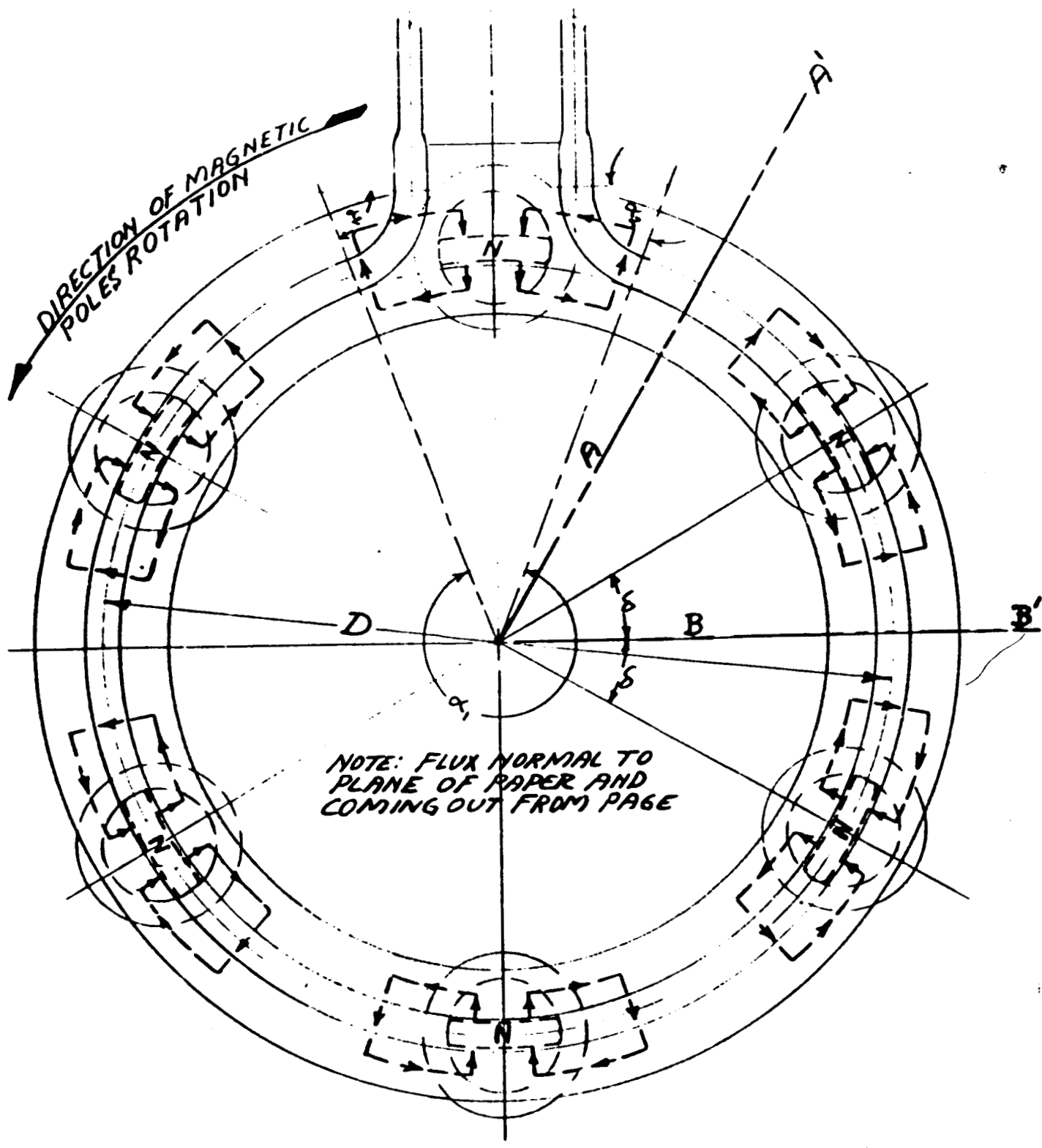
FIGURE 1

PUMP CELL, PLAN VIEW, WITH PROJECTIONS
OF MAGNETIC POLE FACES ON ONE
SIDE OF CELL

PUMP CELL TUBE
CONTAINING LIQUID
METAL.

INNER ELECTRICAL CONDUCTOR

NOTE: FLUX NORMAL TO
PLANE OF PAPER AND
COMING OUT FROM PAGE

OUTER ELECTRICAL CONDUCTOR

POLE FACE

-47-

FIGURE 2

SECTION AA', BB' OF FIGURE 1
EXPLODED, AND THE AVERAGE
PATH OF ONE OF THE 2n EDDY
CURRENTS INDICATED WITH
DIMENSIONS SHOWN IN SUBSTANCE
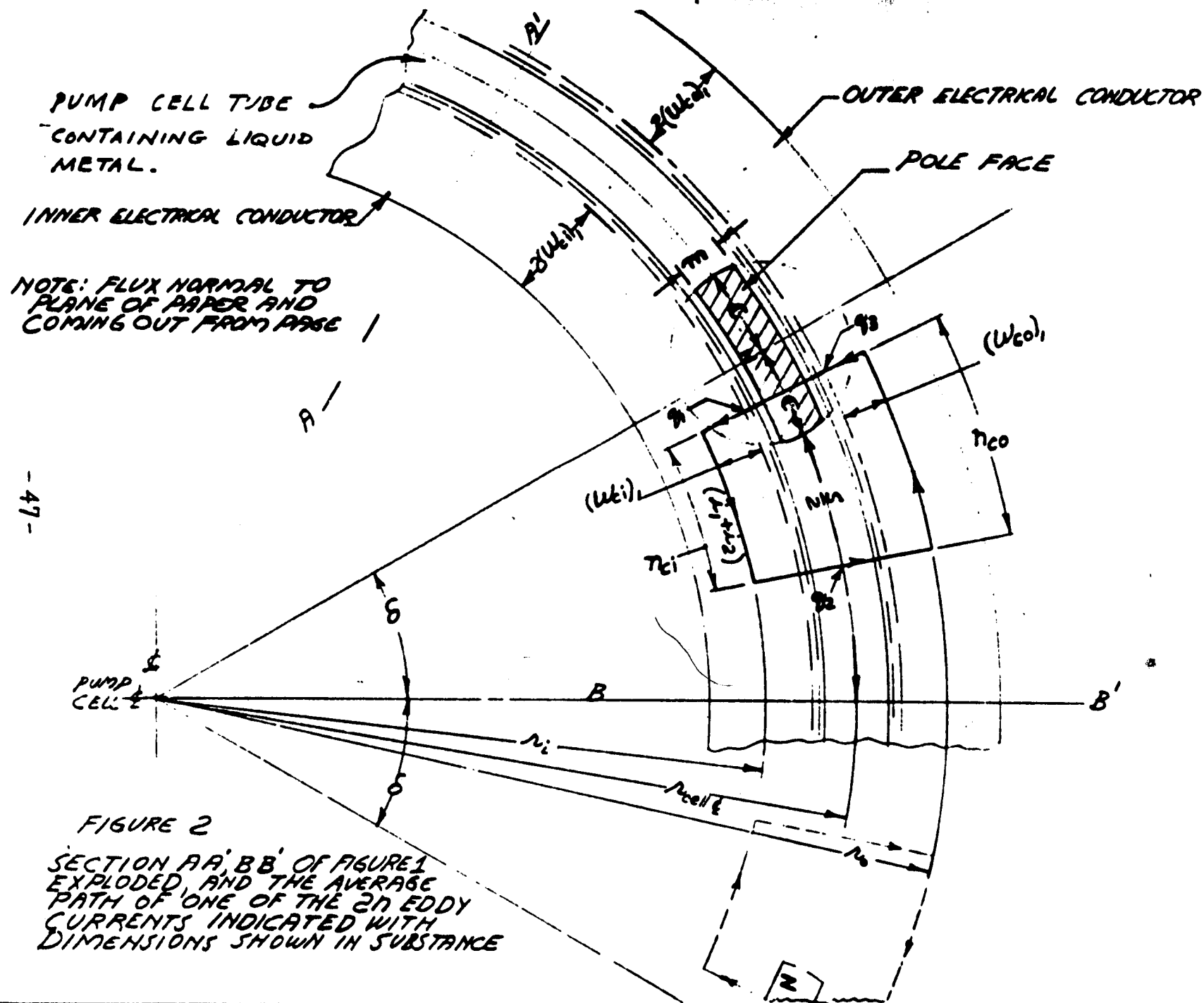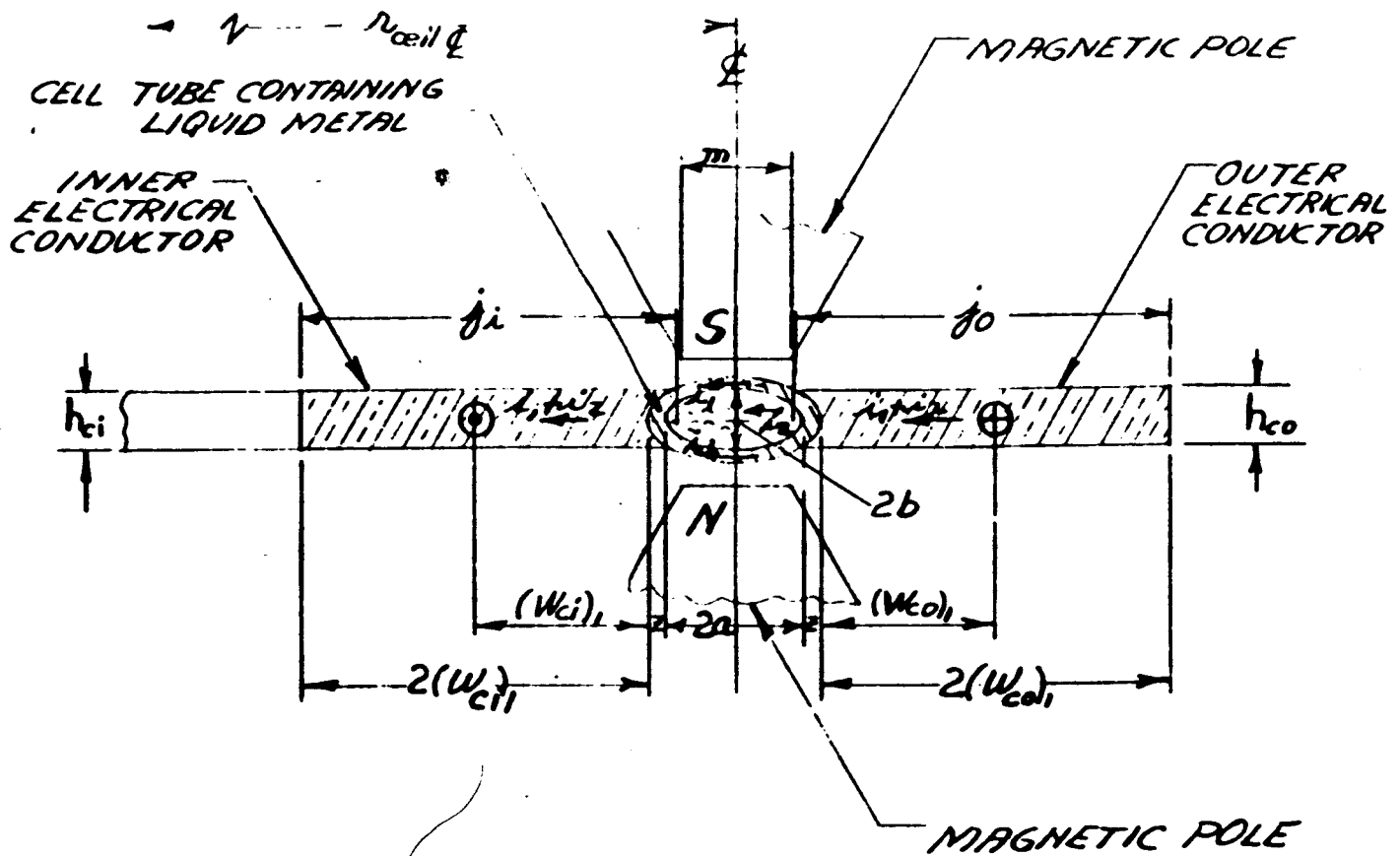
FIGURE 3

SECTIONAL VIEW OF PUMP CELL SHOWN BETWEEN ONE PAIR OF OPPOSED MAGNETIC POLES; AVERAGE EDDY CURRENT PATH AGAIN INDICATED

FIGURE 4

SIMPLIFYING ASSUMPTIONS MADE IN CALCULATING THE
LENGTH OF THE AVERAGE EDDY PATH THROUGH THE LIQUID
METAL, THROUGH THE TUBE WALL BETWEEN THE LIQUID
METAL AND THE CONDUCTOR BARS, AND RADIALLY OUT
THROUGH THE CONDUCTOR BARS
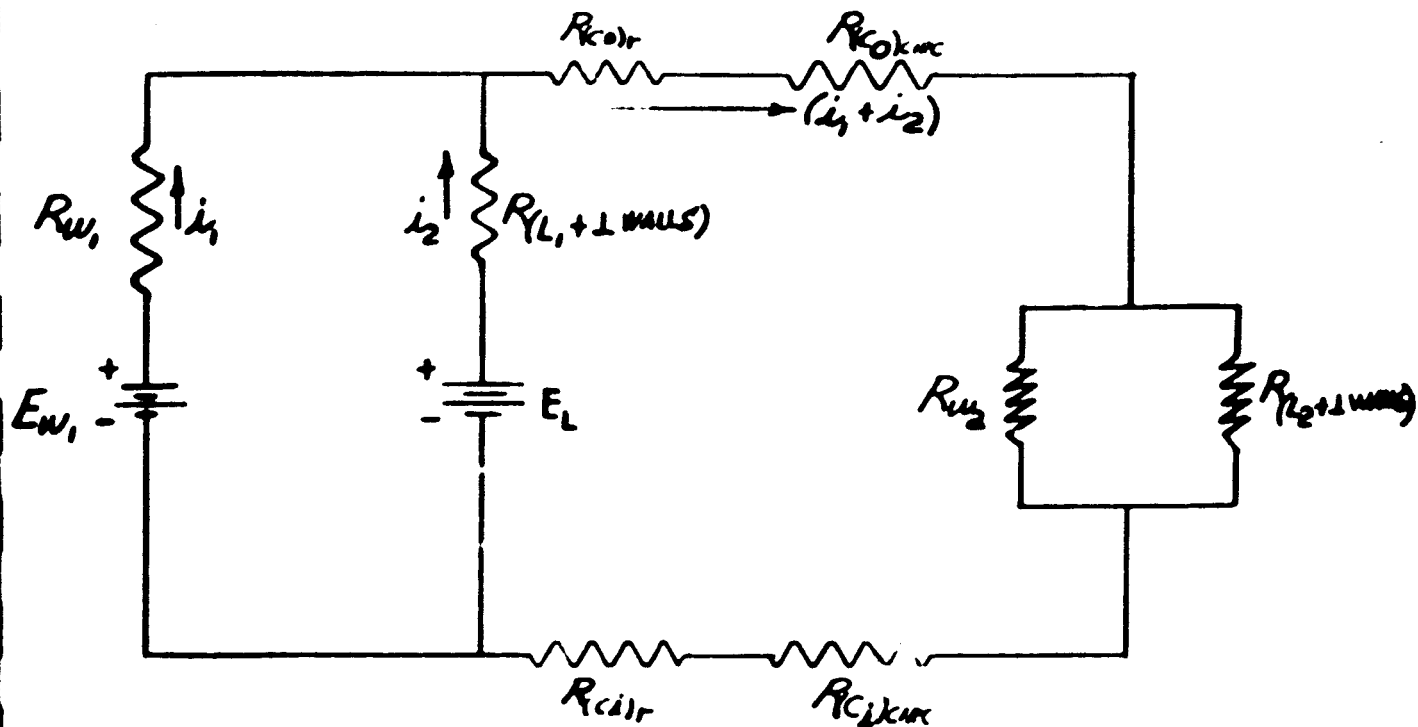
FIGURE 5

EQUIVALENT ELECTRICAL CIRCUIT ASSOCIATED
WITH THE FLOW OF EACH EDDY CURRENT

Note: Path from q_1 along through q_2 to q_3 is
RETURN eddy path, as contrasted to the
path of the eddy through the liquid metal
and walls of the containing tube.

2.1.1.3

Figure 3 shows a sectional view of Pump Cell between
one pair of opposed magnetic poles, with the eddy
current path again indicated.

2.1.1.4

Figure 4 indicates the simplifying assumptions made in
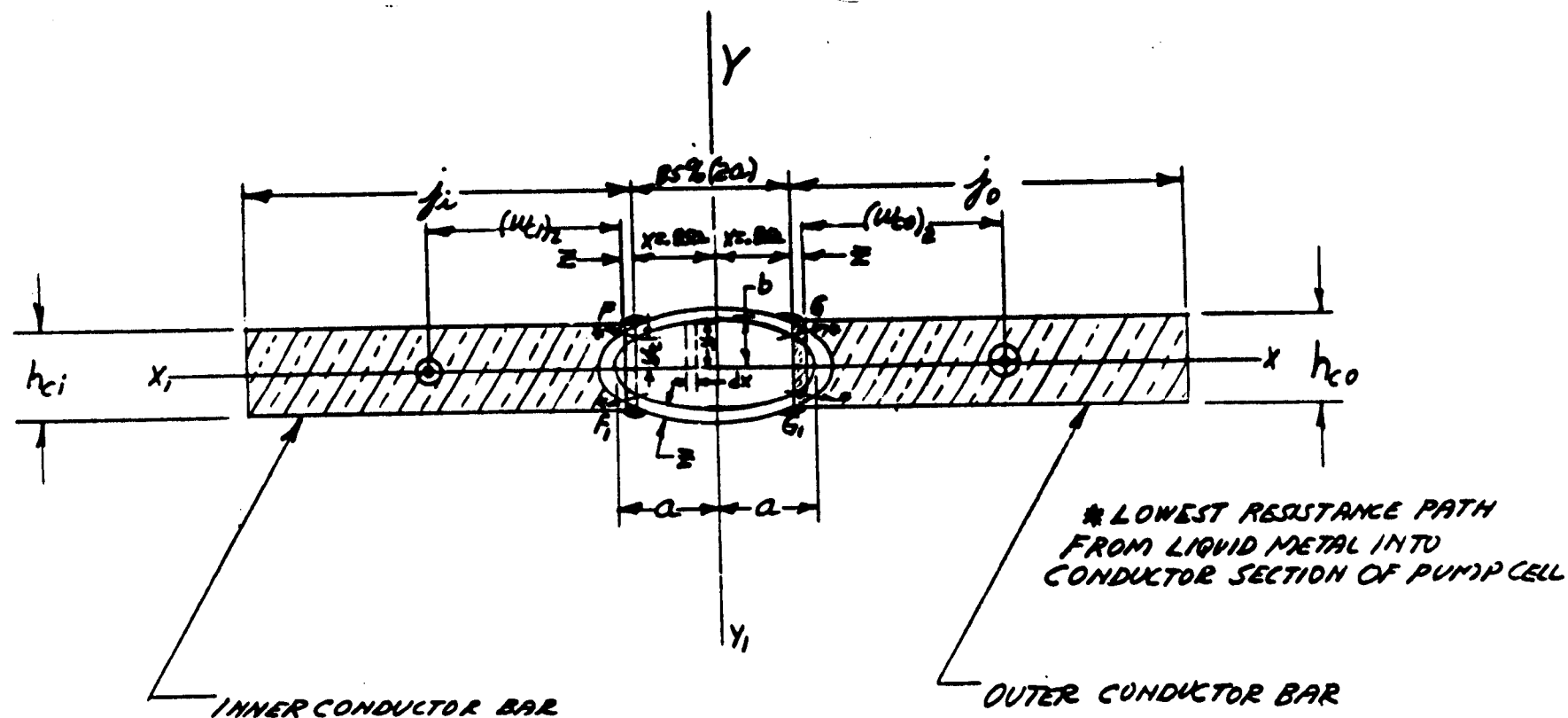calculating the length of the average eddy path through
the liquid metal, through the tube wall between the liquid
metal and the conductor bars, and radially out through
the conductor bars.

2.1.1.4.1

Assume tube walls near ends of major axis
shrunken back along major axis to positions
shaded in at FF' and GG' respectively. This
assumption avoids a complex calculation
involving an integration between the X axis and
the perimeter of the ellipse in determining the
cross section of the path through which the eddy
current will pass in penetrating the wall between
FF' (or GG') and the conductor bar beyond. The
assumption is a good approximation because the
eddy current in these narrowed parts of the
(elliptical) tube will flow in the direction of ——*
in each case rather than take the higher
resistance path to the position $x = a$ and then
out through the tube wall into the conductor bars
(note that the liquid metal being circulated has
a higher resistivity than the material of the
conductor bars).

2.1.1.4.2

The resistance of the liquid metal to the flow of
the eddy current is the resistance imposed by
the liquid metal contained in one quadrant
defined by $x = 0$ to $x = 0.85$ a, $y = 0$ to
$y = y_{terminal} = y_t$, since of the four equal
quadrants of the ellipse two are in series, and
together these two are in parallel to the 3rd and
4th which are themselves in series. The
resistance through one quadrant. which is then
also the total resistance across the liquid metal
path, is given by

51.

$$R = \frac{(\sigma_{LM})(\Delta x)}{yc} \doteq \frac{(\sigma_{LM})}{c} \sum_{x=0}^{x=.85a} \frac{\Delta x}{y} \; ,$$

where

$$y = \frac{b}{a} \sqrt{a^2 - x^2}$$

σ = resistivity, at operating temperature, of the liquid metal being pumped

$$= \frac{(\sigma_{LM})(a)}{bc} \int_{x=0}^{x=.85a} \frac{dx}{\sqrt{a^2 - x^2}}$$

$$= \frac{(\sigma_{LM})(a)}{bc} \left[\sin^{-1} \frac{x}{a} \right]_{x=0}^{x=0.85a}$$

$$= \frac{(\sigma_{LM})(a)}{bc} \; , \; \text{ohms}$$

when a, b, c in inches, σ_{LM} in ohm-in

2.1.1.4.3
Then

(1) Average length of eddy path through liquid metal = (0.85)(2a) (this figure is used only to determine 3, 4 below), inches.

(2) Average length of eddy path through tube wall bounding the liquid metal at ends of major axis of the ellipse = z, inches.

(3) Average length of eddy path radially through outer conductor bar = $(w_{co})_2$ *4

$$= 0.5 (j_o - 0.15a - z) + .15a$$

$$= 0.5 (j_o + 0.15a - z), \text{ inches}$$

(4) Average length of eddy path radially through inner conductor bar = $(w_{ci})_2$

$$= 0.5 (j_i + 0.15a - z), \text{ inches}$$

Note *4: The corresponding dimension $(w_{co})_1$ given on Fig. 3 instead of being measured from the shrunken-back wall, as is the case with $(w_{co})_2$, is measured from the actual physical tube wall at the outside edge of its major axis. This dimen. $(w_{co})_1$ is used to establish the average eddy path circumferentially through the conductor member (see O O on Fig. 3) by means of the modification to $(w_{co})_2$ shown in Fig. 4.

2.1.2 The Equivalent Electrical Circuit Associated with the Flow of Each Eddy Current is shown by Figure 5, in which for the single average eddy current shown in Figure 2

R_{w_1} = combined resistance of both (in parallel) liquid metal-containing tube walls bridging between the two electrical conductor elements (or bars) of the cell and under the magnetic poles, ohms

E_{w_1} = potential generated in the liwuid metal-containing tube walls bridging across the electrical conductors and under the magnetic poles, volts

i_1 = electrical eddy current flowing through liquid metal-containing tube walls bridging between the electrical conductors and under the magnetic poles, amps

i_2 = electrical eddy current flowing through the liquid metal in the area between an opposed pair of magnetic poles (i. e. , directly under a magnetic pole face), amps

$R_{(L_1 + L_{walls})}$ = electrical resistance through the liquid metal and through the portion of the tube walls at each end of the path adjacent to the conductor bars and under the magnetic poles (see Fig. 4), ohms

E_L = electrical potential generated in the liquid metal by rotation of the magnetic pole pairs, volts

$R_{co} = (R_{co})_r + (R_{co})_{circ.}$ = electrical resistance to eddy current through outer conductor element (Note: $R_{(co)_r}$ represents radial component resistances through outer conductor member, while $R_{(co)_{circ.}}$ is circumferential component through the outer conductor member, ohms

$$R_{ci} = R_{(ci)_r} + R_{(ci)_{circ.}} =$$

= electrical resistance to eddy current through inner conductor element (Note: $R_{(ci)_r}$ represents radial components through inner conductor member, $R_{(ci)_{circ.}}$ is circumferential component through inner conductor member, ohms

R_{w_2} = combined electrical resistance of both liquid metal-containing tube walls (in parallel) bridging the two electrical conductor elements of the cell in the return path (see definition under Fig. 2) of the eddy current through the space not covered by magnetic poles, ohms

$R_{(L_2 + \perp walls)}$

= electrical resistance through the liquid metal and through the portion of the tube walls at each end of the electric current path adjacent to the conductor bars and not under magnetic poles, ohms

$(i_1 + i_2)$

= total electric current flowing through the electrical conductors and through the liquid metal and containing tube walls in the return eddy path through the liquid metal lying outside the magnetic flux field, amps

2.1.3 Basic Equations Used in Calculating Electrical Parameters (Refer to Fig. 5)

2.1.3.1
$$E_{w_1} - i_1 R_{w_1} = E_L - i_2 R_{(L_1 + \perp walls)}$$

2.1.3.2
$$E_L - i_2 R_{(L_1 + \perp walls)} =$$

$$= -\left[-(i_1 + i_2)(R_{co} + \frac{R_{w_2} \times R_{(L_2 + \perp walls)}}{R_{w_2} + R_{(L_2 + \perp walls)}} + R_{(ci)} \right]$$

2.1.3.3

$$R = \frac{\sigma \, \ell}{A} \text{ ohms}$$

where

R = resistance to be calculated, ohms
ℓ = length of path along which eddy current flows, in.
A = cross sectional area through which eddy current flows, sq. in.
σ = resistivity of material through which eddy current flows, ohm - in.

2.1.3.4

$$R_{w_2} = R_{w_1} \times \frac{c}{s/2} \text{ ohms}$$

2.1.4 Electrical Resitance through Portion of Path through Liquid Metal between Magnetic Poles of Opposite Polarity

$$R_{L_1} = \frac{\sigma \, a}{b \, c} \qquad \text{(for delineation, see Fig. 4)} \quad \text{ohms}$$

where

σ = resistivity of liquid metal (at operating temperature), ohm inches
a = 1/2 major axis of tube interior, inches
b = 1/2 minor axis of tube interior, inches
c = 1/2 length of magnetic pole face in direction along line of magnet rotation (see Fig. 1), inches

2.1.5 Electrical Resitance through Walls between the Liquid Metal and the Inner and Outer Conductors in Portion of Eddy Path Lying between Magnetic Poles of Opposite Polarity (see Fig. 4)

$$R_{(\perp \text{ walls under magnetic poles})} = \frac{(\sigma_{\text{tube mat'l}}) \, (z)}{(y_t) \, (c)} \text{ ohms}$$

where

σ = resistivity of material of tube walls (at operating temperature), ohm inches
z = Thickness of tube wall, inches

$$y_t = \frac{b}{a} \sqrt{a^2 - (0.85)^2 \, (a)^2} \quad , \text{ inches (see Fig. 4)}$$

where

if tube is elliptical in section

b = 1/2 minor axis of tube interior section, inches

a = 1/2 major axis of tube interior section, inches

and, if tube is round in interior section,

a = b = 1/2 I. D. of tube, inches

c = same as under 2.1.4 above

2.1.6 Combining 2.1.4 and 2.1.5, the electrical resistance through both the liquid metal and the portions of the tube walls attached to the inner and outer conductors =

$$R_{(L_1 + \perp walls)} = R_{L_1} + R_{(\perp walls \ under \ magnetic \ poles)}, \ ohms$$

2.1.7 Electrical Resistance through the Liquid Metal and Portion of Tube Walls Attached to the Inner and Outer Conductors in the RETURN Eddy Path

$$\text{Equivalent Resistance} = \frac{R_{w_2} \times R_{(L_2 + \perp walls)}}{R_{w_2} + R_{(L_2 + \perp walls)}}, \ ohms$$

where

$$R_{(L_2 + \perp walls)} = R_{(L_1 + \perp walls)} \times \frac{c}{s/2}$$

where

$R_{(L_1 + \perp walls)}$ is given in 2.1.6 above

c is as in 2.1.4 above

$s/2$ is as defined under Fig. 2, and =

$$= (1/2) (\pi D/n) - c$$

where

n = number pole pairs, or poles of same polarity

D = diameter of centerline of liquid metal-containing tube (see Fig. 1), inches

c = same as in 2.1.4 above

56.

2.1.8 Electrical Resistance in Circumferential Direction

2.1.8.1   Through Outer Circumferential Conductor Bar:

$$R_{\substack{\text{circumferential} \\ \text{outer conductor bar}}} = \frac{(\sigma_{\text{conductor mat'l}}*)(n_{co})}{(0.98)(j_o - z)(h_{co})} \text{ , ohms}$$

* at operating temp.

where

$\sigma_{\text{conductor mat'l}}$ = resistivity of that mat'l (at operating temperature), ohm inches

n_{co} = as defined in Figure 2, inches

j_o = as defined in Figures 3, 4, inches

h_{co} = as defined in Figures 3, 4, inches

z = wall thickness of tube containing liquid metal, inches (figures 3 & 4)

0.98 factor: area of conductor normal to circumferential path of eddy current is a little smaller than $(j_o - z)(h_{co})$ because of groove cut out for LM-containing tube; therefore, area arbitrarily reduced by 2%.

2.1.8.2   Through Inner Circumferential Conductor Bar:

$$R_{\substack{\text{circumferential} \\ \text{inner conductor bar}}} = \frac{(\sigma_{\text{conductor mat'l}}*)(n_{ci})}{(0.98)(j_i - z)(h_{ci})} \text{ , ohms}$$

* at operating temp.

where

$\sigma_{\text{conductor mat'l}}$ is as in 2.1.8.1 above

n_{ci} is as defined in Figure 2, inches

j_i is as defined in Figures 2 & 3, inches

h_{ci} is as defined in Figure 3, inches

b, a, are as under 2.1.8.1 above, inches

z = wall thickness of tube containing liquid metal, inches

0.98 factor: as under 2.1.8.1, above

2.1.8.3   Note: if conductor material is in any way clad, use inside dimensions of cladding to obtain w_{co}, w_{ci}, h_{co}, h_{ci}.

57.

2.1.9 Electrical Resistance in Radial Direction

2.1.9.1 Through Outer Circumferential Conductor Bar, $R_{(co)_r}$

$$R_{\substack{\text{radial outer} \\ \text{conductor bar}}} = R_{(co)_{r_{\substack{\text{adjacent to} \\ \text{magnetic poles}}}}} + R_{(co)_{r\text{RETURN path}}}$$

where
  "co" refers to Outer Conductor
  "r" refers to radial direction

$$R_{(co)_{r\text{RETURN path}}} = R_{(co)_{r_{\substack{\text{adjacent to} \\ \text{magnetic poles}}}}} \times \left(\frac{c}{s/2}\right)$$

and c, $s/2$ are as defined by Figure 2.

$$R_{\substack{\text{radial outer} \\ \text{conductor bar}}} = R_{(co)_{r_{\substack{\text{adjacent to} \\ \text{magnetic poles}}}}} + R_{(co)_{r_{\substack{\text{adj. to} \\ \text{mag. poles}}}}} \times \left(\frac{c}{s/2}\right)$$

$$= R_{(co)_{r_{\substack{\text{adj. to} \\ \text{mag. poles}}}}} \left(1 + \frac{c}{s/2}\right)$$

where
$$R_{(co)_{r_{\substack{\text{adj. to} \\ \text{mag. poles}}}}} = \frac{(\sigma_{\text{cond. mat'l}})(w_{co})2}{(c)(h_{co})}$$

Refer to Fig. 4 for identification of $(w_{co})_2$, h_{co} ; to Fig. 2 for c.

2.1.9.1.1 Path adjacent to magnetic poles is indicated in Figure 2 by radial line through q_3 , q_1 .

2.1.9.1.2 RETURN path of eddy current is indicated in Figure 2 by radial line through q_2 .

2.1.9.2 Through Inner Circumferential Conductor Bar, $R_{(ci)_r}$

$$R_{(ci)_r} = R_{(ci)_{r_{\substack{\text{adj. to} \\ \text{mag. poles}}}}} + R_{(ci)_{r\text{RETURN path}}}$$

where
$$R_{(ci)_{r\text{RETURN path}}} = R_{(ci)_{r_{\substack{\text{adj. to} \\ \text{mag. poles}}}}} \times \left(\frac{c}{s/2}\right)$$

and c, $s/2$ are as defined by Figure 2.

$$= R_{(ci)_{r_{adj.\ to\ \oint\ mag.\ poles}}} + (R_{(ci)_{r_{adj.\ to\ mag.\ poles}}})(\frac{c}{s/2})$$

$$= R_{(ci)_{r_{adj.\ to\ mag.\ poles}}}\ (1\ +\ c/(s/2)\ )$$

where

$$R_{(ci)_{r_{adj.\ to\ mag.\ poles}}} = \frac{(\sigma_{cond.\ mat'l})(w_{co})2}{(c)\ (h_{ci})}$$

2.2 If the pump cell is of double plate construction, so that the material is the same for the liquid metal-containing walls and for the conductor sections on either side of the liquid metal passageway, then:

2.2.1 Relative Physical Dimensions of Cell, and Associated Magnetic Poles, Required for Electrical (Eddy) Current Calculations:

2.2.1.1

In Figure 6 (Pump Cell: Plan View Section, with projections of magnetic pole faces on one side of cell section) and Figure 7 (Section AA'BB' of Figure 6, exploded, and the average path of one side of the n eddy currents indicated with dimensions shown in substance), let

(1) Magnetic poles on one side of tube be alternately of opposite polarity; on the other side of tube, each pole is of polarity opposite to that facing it across the liquid-metal-containing passageway.

(2) Number of magnetic pole pairs = n

(3) Average eddy paths be as indicated by arrows; number of such paths = n

2.2.1.2

Figure 8 shows the Sectional View of Pump Cell between one pair of opposed magnetic poles; average eddy current path again indicated.

San.. simplifying assumptions are made in calculating the length of the average eddy path through the liquid metal, and radially through the conductor bars as

59.

DIRECTION OF MAGNETIC POLES ROTATION

NOTE: FLUX NORMAL TO PLANE OF PAPER AND CUMING OUT FROM PAGE (OR GOING IN).

FIGURE 6

PUMP CELL, PLAN VIEW SECTION, WITH
PROJECTIONS OF MAGNETIC POLE
FACES ON ONE SIDE OF CELL SECTION,
WITH POLARITIES ALTERNATED.
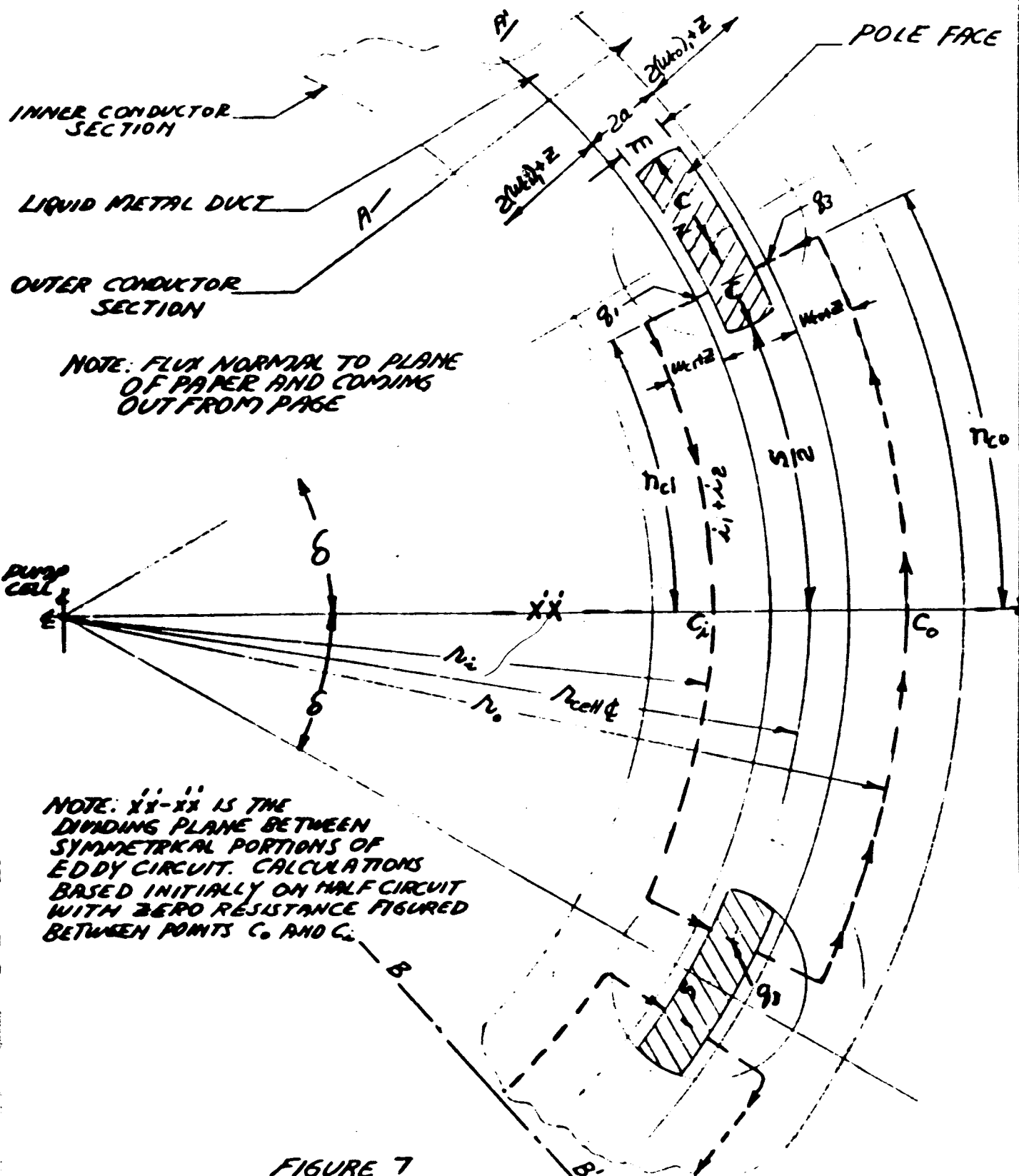
FIGURE 7

SECTION AA', BB' OF FIGURE 6 EXPLODED, AND THE AVERAGE PATH OF ONE OF THE n EDDY CURRENTS INDICATED WITH DIMENSIONS SHOWN IN SUBSTANCE
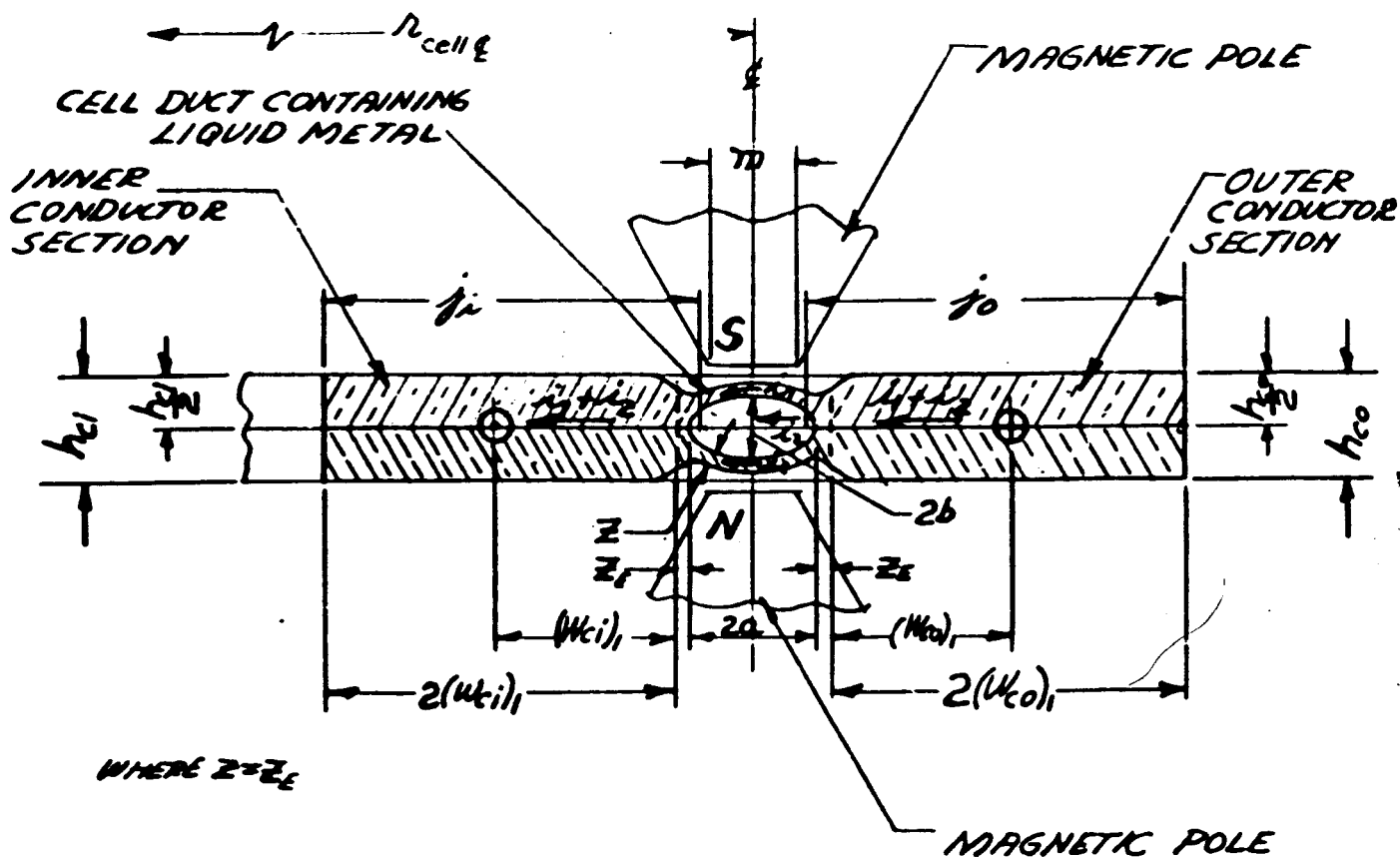
FIGURE 8

SECTIONAL VIEW OF PUMP CELL SHOWN BETWEEN
ONE PAIR OF OPPOSED MAGNETIC POLES; AVERAGE
EDDY CURRENT PATH AGAIN INDICATED
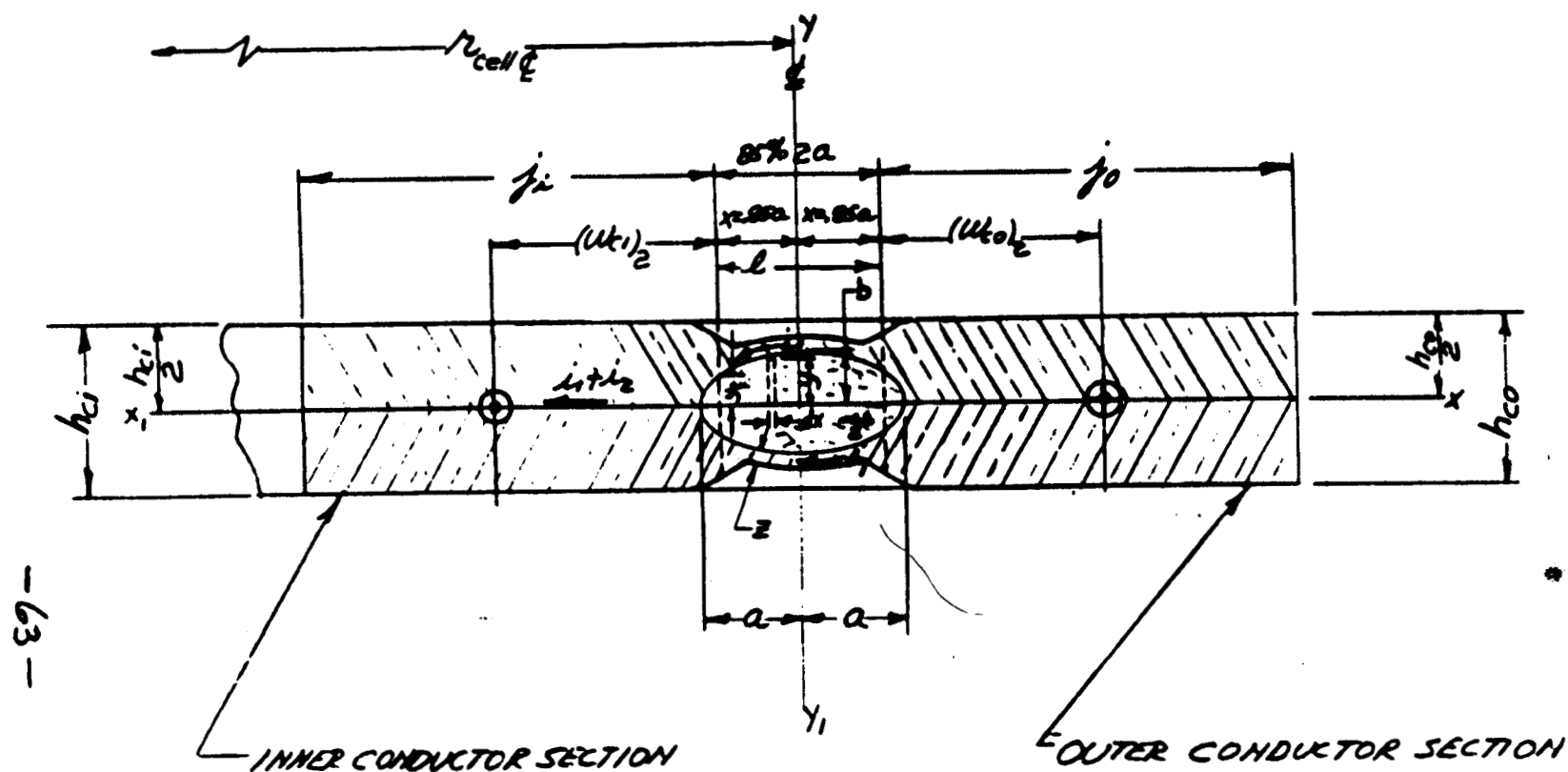
FIGURE 9

SIMPLIFYING ASSUMPTIONS MADE IN CALCULATING THE
LENGTH OF THE AVERAGE EDDY PATH THROUGH THE LIQUID
METAL, AND RADIALLY THROUGH THE CONDUCTOR SECTIONS.
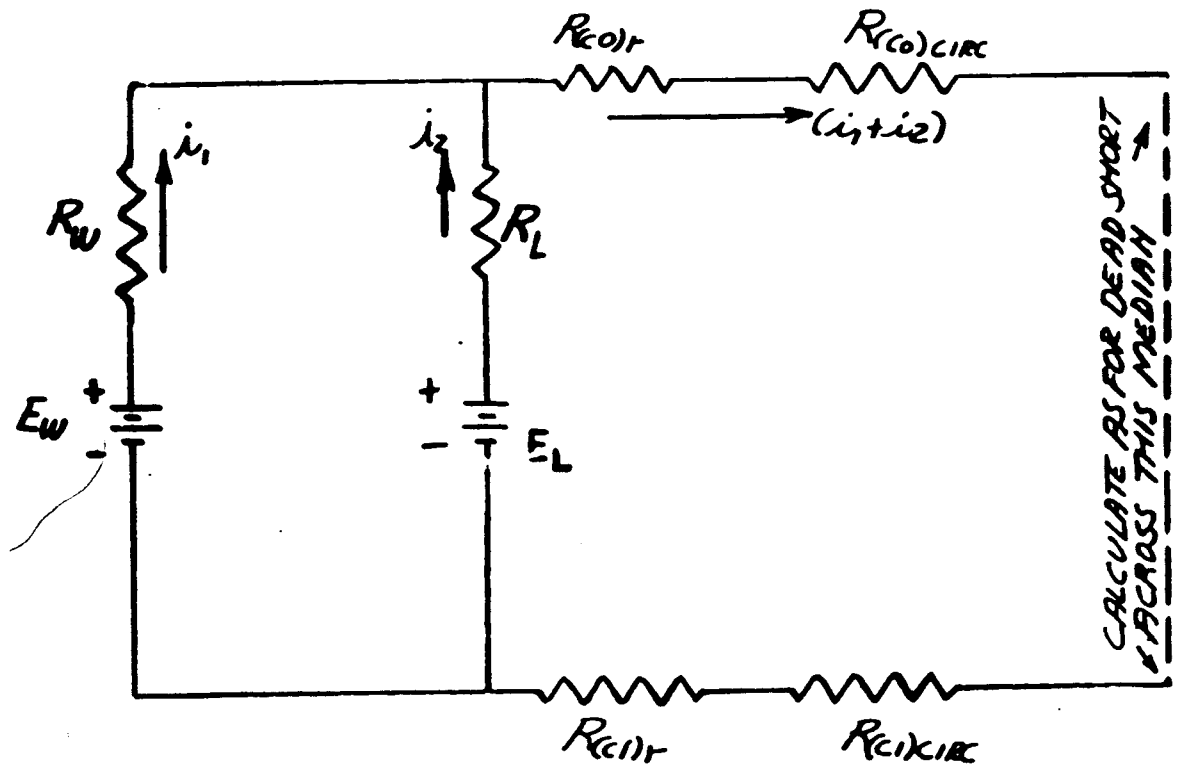
FIGURE 10

EQUIVALENT ELECTRICAL CIRCUIT ASSOCIATED
WITH THE FLOW OF EACH EDDY CURRENT

under 2.1.1.4 except that in the present case there are no tube walls to be considered in the eddy current path from the liquid metal into the conductor section, and it is just the radiused conductor-section walls bounding the liquid metal that are shrunk back for calculating purposes (see Figure 9).

2.2.1.3

Figure 9 shows the simplifying assumptions made in calculating the length of the average eddy path through the liquid metal and radially through the conductor section.
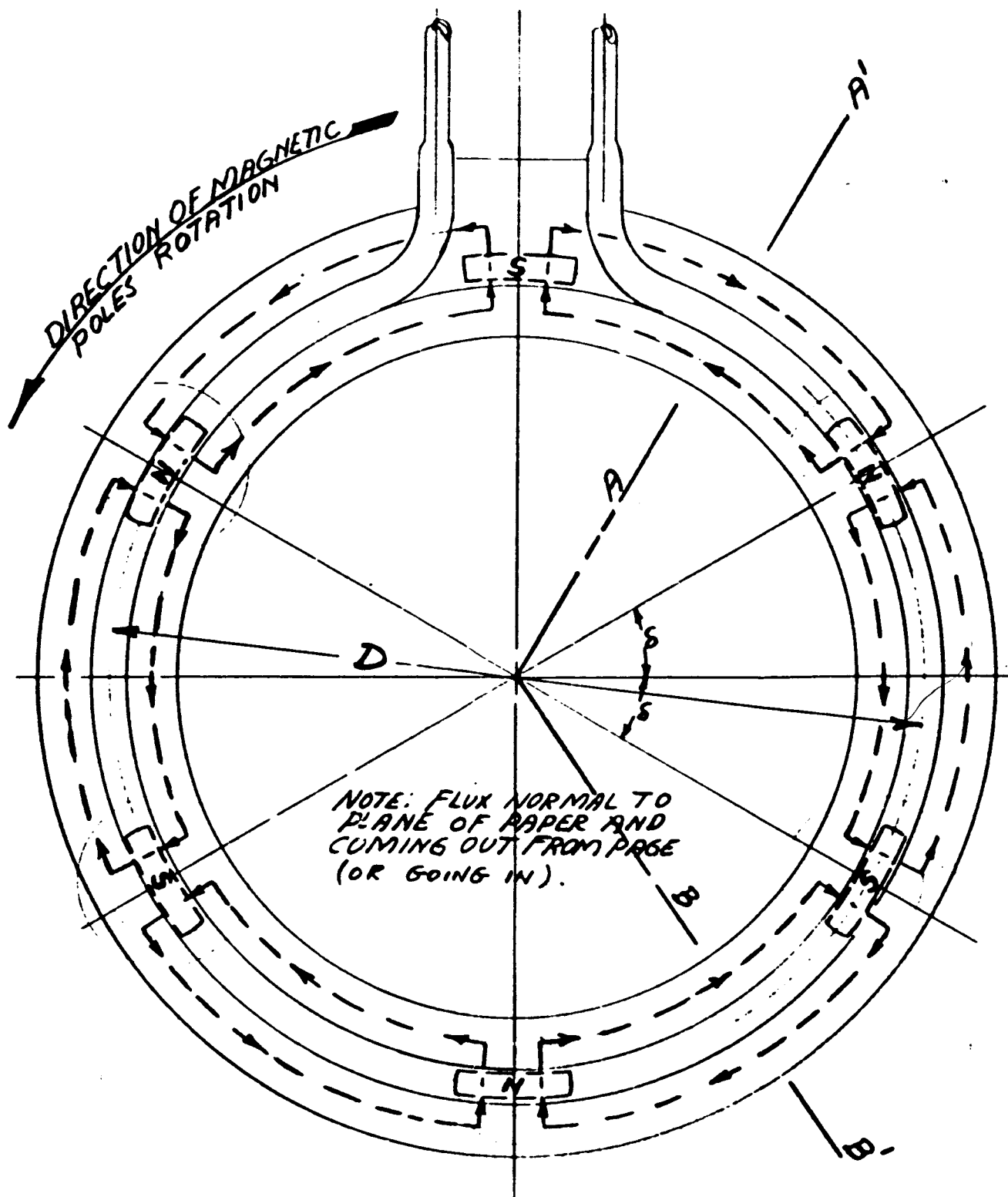
2.2.2

Figure 10 shows the equivalent electrical circuit associated with the flow of an average eddy current (see Fig. 7). Only half the path need be used for calculation purposes, with a dead short circuit (i.e., no electrical resistance) assumed across the cutting line $C_{ci} - C_{co}$ in Fig. 7).

R_{w_1} = combined resistance of both (in parallel) liquid metal-containing walls bridging between the two electrical conductor element (or bars) of the cell and under the magnetic poles, ohms

E_{w_1} = potential generated in the liquid-metal-containing walls bridging across the electrical conductors and under the magnetic poles, volts

i_1 = electrical eddy current flowing through the liquid metal-containing walls bridging between the electrical conductors and under the magnetic poles, amps

i_2 = electrical eddy current flowing through the liquid metal in the area between an opposed pair of magnetic poles (i.e., directly under a magnetic pole face), amps

R_{L_1} = electrical resistance through the liquid metal under the magnetic poles, ohms

E_L = electrical potential generated in the liquid metal by rotation of the magnetic pole pairs, volts

R_{co} = electrical resistance to eddy current through outer conductor element, $= R_{(co)_r} + R_{(co)_{circ.}}$ ohms .

R_{ci} = electrical resistance to eddy current through inner conductor element, $= R_{(ci)_r} + R_{(ci)_{circ.}}$ ohms

$(i_1 + i_2)$

= total electric current flowing through the electrical conductors, amps

65.

2.2.3 Basic Equations Used in Calculating Electrical Parameters
(Refer to Figures 9 & 10)

2.2.3.1

R_w = Combined resistance of the two liquid-metal-containing walls bridging the inner and outer conductor sections of the cell plates (Fig. 9), ohms

$$= 0.5 \, (\sigma_{\text{plate mat'l.}}) \, (\text{Frac. Perimeter})/(z)(c)$$
@ op. temp.

where

0.5 factor: combined resistance of two equal resistance in $//$ = $(1/2)$ the resistance of one of the resistances

Frac. Perimeter:
that part of the ellipse perimeter bounding the sector defined by
x = +0.85a, x = - 0.85a; inches
y = 0; y = + y_t , inches

This arc length may be approximated by the expression
$(1/2)(0.85)$(ellipse perimeter), inches

z = Thickness of elliptical wall (see Figs. 8 & 9), inches

c = $(1/2)$ pole face length (see Fig. 7), inches
Note: If one half the length along cell tube's curved axis is covered by magnetic poles, then the arc length covered by pole is $(1/2)(\pi D_{\mathcal{C} \, \text{cell}})/n$, where n = number of pole pairs

$\sigma_{\text{plate mat'l}}$
@ op. temp.
= resistivity of the plate material comprising the cell; value used must be that in effect at operating temperature, ohm-inches

66.

2.2.3.2

R_{L_1} = Resistance of the liquid metal between one opposed pair of magnetic poles to the flow of the eddy currents (see Figs. 7 & 8; also derivation under section 2.1.1.4.2)

$$= \frac{(\sigma_{LM})\, a}{b\, c} \quad \text{ohms}$$

where

σ = resistivity of liquid metal being circulated, ohm- inches

a = major axis elliptical LM passage section, inches

b = minor axis elliptical LM passage section, inches

c = (1/2) length of magnetic pole face, inches

2.2.3.3

$$R_{(co)_{circ.}} =$$

$$= \frac{(\sigma_{plate\ mat'l})(2)\ (1/2)(\pi/n)(D_{cell\ \mathcal{C}_L} + 2 \times 0.85 \times a + 2(w_{co})2 - (c/2)(r_o/r_{cell\ \mathcal{C}_L})}{0.98\,(j_o)\,(h_{co})}$$

where

$r_{cell\ \mathcal{C}_L}$ = radius from center of pump cell to $\mathcal{C}_L$ of liquid metal passage, inches

n = number of pole pairs

0. 98 factor: area of conductor section is a little smaller than $(j_o)(h_{co})$ - see Fig. 9; a 2% reduction is taken arbitrarily.

See Figures 7 & 9.

2.2.3.4

$$R_{(ci)_{circ.}} =$$

$$= \frac{\sigma_{plate\ mat'l}\ (2)\ (1/2)(\pi/n)\ (D_{cell\ \mathcal{C}_L} + 2 \times 0.85a + (2)(w_{ci})2 - (c/2)(r_i/r_{cell\ \mathcal{C}_L})}{0.98\,(j_i)\,(h_{ci})}$$

2.2.3.5

$$R_{(co)_r} = \frac{(\sigma_{\text{plate mat'l}})(w_{co})2}{(h_{co})(c)(r_o/r_{\text{cell}} \; \mathbb{C})} \quad \text{ohms}$$

2.2.3.6

$$R_{(ci)_r} = \frac{(\sigma_{\text{plate mat'l}})(w_{co})2}{(h_{ci})(c)(r_{ci}/r_{\text{cell}} \; \mathbb{C})} \quad \text{ohms}$$

2.2.3.7

General rule of LM present design practice is that $\Sigma R_{\text{conductor}}$ should be approximately $1/3 \; \Sigma \; \text{Resistance}_{\text{Eddy Path}}$. In case of high resistivity liquid metals such as bismuth, this fraction may be only $1/6$.

2.2.4 Electric Current and Potential Values for Equivalent Circuit of 1/2 Eddy Path <u>**for nominal flow**</u> and **pressure**

2.2.4.1

$$F = 0.57 \; B \; i_2 \, \ell \times 10^{-6} \, , \quad \text{lbs}$$

where

F = force required per 1/2 eddy being calculated

$$= \frac{\Sigma \text{ force generated}}{\text{number } 1/2 \text{ eddies}}$$

$$= \frac{(\text{nominal developed pressure head} + \text{hydraulic losses})(A)}{2 \; n}$$

where

n = number opposed pole pairs in magnetic rotor assembly

nominal developed pressure head is known in each case (design conditions)

hydraulic losses are given in Section 1.4

A = sectional area interior liquid metal passageway for circular passage = π (I. D.) $/4$; for elliptical passage = π a b, where <u>a</u> and <u>b</u> are lengths of semi-major and semi-minor axis respectively, square-inch

B $\quad=$ Magnetic flux density across the magnetic pole gap, in gauss. This value depends on the length of the air gap between the pole pairs, and may be taken from a curve of B vs. air gap for a particular design of pump, or it may be assumed a reasonable value and then altered as required.

i_2 $\quad=$ average eddy current flowing through the liquid metal, in amps. This is an unknown and will be solved for.

ℓ $\quad=$ (0.85)(2a), in inches (see Fig. 9)
reason for 0.85 factor is given under Sections 2.1.1.4.3 and 2.1.1.4.1

and, rearranging,

$$(i_2)_{nom} = \frac{F}{0.57 \, B \, \ell \times 10^{-6}} \quad \text{amps}$$

2.2.4.2 From the basic equation for generation of electric potential, 2.2.4.2.1, the electric potential generated in the plate walls bridging the conductor portions (inner and outer heavy section of plate) is

2.2.4.2.1

$$(E_w)_{nom} = B(2.54 \, \ell)(30.5 \, V_R) \times 10^{-8} \quad \text{volts}$$

where ℓ is the same as under 2.2.4.1, because the projected dimension rather than the arc of the bridging wall between the conductor portions of the plate section (see Fig. 9) is required.

. . . . 2.2.4.2.2

Then the electrical potential generated in the liquid metal, at nominal (i.e., design) conditions, is

$$(E_L)_{nom} = (E_w)_{nom} \times \frac{V_R - (V_L)_{nom}}{V_R} \quad \text{volts}$$

where

V_R is the rotor velocity, ft sec.

V_L is the velocity of the liquid metal, ft sec. (determined from capacity of pump).

$$= \boxed{B(2.54 \, \ell)(30.5) \times 10^{-8}} \, V_R - (V_L)_{nom}$$

69.

2.2.4.2.3 Refering to Figure 10,

$$(E_w)_{nom} - (i_1)_{nom} R_W = (E_L)_{nom} - (i_2) R_L$$

where

$(i_1)_{nom}$ = eddy current flowing through the plate walls bridging between the inner and outer conductor portions of the plate. (see Figures 8 & 9)

therefore,

$$(i_1)_{nom} R_w = (E_L)_{nom} + (i_2)_{nom} R_L$$

and

$$(i_1)_{nom} R_w = \frac{(E_w)_{nom} - (E_L)_{nom} + (i_2)_{nom} R_L}{R_w} \text{ amps}$$

2.2.4.2.4 Refering again to Figure 10, by Kirchoff's Rule,

$$(E_w)_{nom} - (i_1)_{nom} R_w =$$

$$= - (i_{1_{nom}} - i_{2_{nom}})(R_{co_{circ}} + R_{co_r} + R_{ci_{circ}} + R_{circ})$$

in which, by the foregoing calculations, all values are known numerically except $(E_w)_{nom}$ which

from Section 2.2.4.2 is known only in terms of V_R;

therefore, V_R in ft/sec. is determined now from

this equation, and can be converted to rpm by

multiplying by $(60/2 \; r_{cell} \; \mathcal{C}_L)$.

2.2.4.3

The slip velocity, V_S, of the circulating liquid metal with respect to the rotating magnetic structure is given at nominal operation conditions by

$$V_S = V_R - (V_L)_{nom} , \text{ in ft/sec.}$$

where V_R and $(V_L)_{nom}$ are both known in ft/sec.

Now both $(E_w)_{nom}$ and $(E_L)_{nom}$ can be determined numerically.

70.

III. CALCULATION OF ELECTRODYNAMIC PUMP PERFORMANCE CURVE

3.1 **Calculations for Blank-off** (maximum pressure head, zero liquid metal flow)

3.1.1

$$(V_L)_{C=0} = 0$$

where $\underline{C}$ is the capacity of the pump @ blank-off, in gpm (here $C=0$)

therefore,

$$V_R = (V_S)_{C=0}$$

3.1.2

$$(E_w)_{C=0} = (E_L)_{C=0} \quad ; \text{ also}$$

E_w is constant for any given rotor speed

therefore,

$$(E_w)_{C=0} = (E_w)_{nom.} \text{ , which was calculated under}$$

2.2.4.3 above.

3.1.3

$$(i_1)_{C=0} = \frac{(E_w)_{C=0} - (E_L)_{C=0} + (i_2)_{C=0} \ R_L}{R_w}$$

$$= (i_2)_{C=0} \ (R_L/R_w) \text{ , amps}$$

which will give $(i_1)_{C=0}$ in terms of $(i_2)_{C=0}$, also unknown at this stage of the calculations.

3.1.4

Similar to 2.2.4.2.4 above,

$$(E_w)_{C=0} - (i_1)_{C=0} \ R_w \doteq$$

$$- \left[-(i_1)_{C=0} - (i_2)_{C=0} \right] \left[R_{co_{circ}} + R_{co_r} + R_{ci_{cir}} + R_{ci_r} \right]$$

where only $(i_2)_{C=0}$ is unknown and therefore may be solved for, the answer being in amps when resistances are in ohms, and potentials in volts.

3.1.5

Similar to 2.2.4.1,

$$(F)_{C=0} = 0.57 \, B \, (i_2)_{C=0} \, \ell \, \times \, 10^{-6} \, , \, \text{lbs}$$

where $(i_2)_{C=0}$ is known from 2.2.5.4 above, and B and ℓ are known from Section 2.2.4.1

then, Effective pressure = $\dfrac{(F)_{C=0} \, (2n)}{A}$ psi
(pressure generated)$_{C=0}$

where

n = number opposed pole pairs
A = cross sectional area liquid metal flow passage, square inch

3.2 Calculations for Free Flow (maximum liquid metal flow, zero effective pressure).

3.2.1

Hydraulic loss = generated pressure head

3.2.2

$$P_{(C = max)} = \frac{2 \, n}{A} \, (F_{C=max})$$

where

n = number pairs of poles
A is as defined under Section 3.1.5

$$= \frac{2 \, n}{A} \, (0.57 \, B \, (i_2)_{C=max} \, \ell \times 10^{-6})$$

where

$P_{(C=max)}$ and $(i_2)_{C=max}$ are both unknown.

3.2.3

Slip Velocity at free flow = $(V_S)_{C=max} = V_R - (V_L)_{C=max}$

$$= V_R - (\frac{C_{max}}{C_{nom}})(V_L)_{nom}$$

3.2.4

$$(E_L)_{C=max} = (E_w)_{constant} \frac{(V_S)_{C=max}}{V_{R_{constant}}}$$

$$= (E_w)_{constant} \frac{V_R - (V_L)_{nom} (C_{max} / C_{nom})}{V_R}$$

where

V_R is a constant, and is the same value as determined under 2.2.5.1 above.

72.

3.2.4 (cont'd)

$$(E_w)_{C=max} = (E_w)_{nom} \text{, and is the same value}$$
$$\text{as calculated under 2.2.4.3 above.}$$

C_{max} is unknown; therefore $(E_L)_{C=max}$ is expressed here only in terms of C_{max}.

3.2.5

$$(E_w)_{constant} - (i_1)_{C=max} R_w = (E_L)_{C=max} - (i_2)_{C=max} R_L$$

where

$(i_1)_{C=max}$ is unknown

$(E_L)_{C=max}$ is defined (see Section 2.2.6.4) in terms of C_{max}

$(i_2)_{C=max}$ is unknown

Solve for $(i_1)_{C=max}$ in terms of C_{max} and $(i_2)_{C=max}$

3.2.6

Similar to 3.1.4 above

$$(E_w)_{constant} - (i_1)_{C=max} R_w =$$

$$= - \left[-(i_1)_{C=max} - (i_2)_{C=max} \right] \left[R_{co_{circ}} + R_{co_r} + R_{ci_r} \right]$$

Substitute for $(i_1)_{C=max}$ the value, obtained under 2.2.6.5 above, in terms of C_{max} and $(i_2)_{C=max}$; the value $(i_2)_{C=max}$ remains as an unknown.

Solve for $(i_2)_{C=max}$ in terms of C_{max}

3.2.7

Now, from 3.2.2 ,

$$P_{C=max} = \frac{2n}{A} \left[\overline{0.57\ B\ (i_2)_{C=max}\ \ell \times 10^{-6}} \right]$$

But also,

$$P_{C=max} = \text{hydraulic loss @ } C_{max} \quad (\text{see Section } 2.2.6.1)$$

$$= (\text{hydraulic loss})_{nom}\ (C_{max}/C_{nom})^2$$

Therefore,

$$\frac{(C_{max})^2}{(C_{nom})^2}\ (\text{hydraulic loss})_{nom} = \frac{2n}{A} \left[\overline{0.57 B(i_2)_{C=max}\ \ell \times 10^{-6}} \right]$$

Substitute for $(i_2)_{C=max}$ the value, in terms of C_{max} , determined under 2.2.6.6 above.

This then gives a quadratic equation with only one unknown, i.e., C_{max}, which can be solved by the standard method

$$C_{max} = \frac{-b \overset{+}{\underset{-}{}} \sqrt{b^2 - 4ac}}{2a} \quad \text{gpm}$$

where a, b, c are as generally defined for the solution of a quadratic equation.

3.3 Three points of the calculated performance curve are now at hand:

3.3.1　The nominal conditions of flow rate (gpm) and required developed pressure head (psi).

3.3.2　The blank-off conditions of zero flow (gpm) and a maximum pump pressure (psi) as found under section 2.2.5.5.

3.3.3　free flow conditions of zero effective pressure (psi) and the maximum pump capacity (gpm) as calculated under Section 2.2.6.7 above.

The pump performance curve should now be plotted as pressure (ordinate) vs. capacity (abscissa).

74.

3.4 To establish the correctness of the curve drawn through the three points specified above, a check point is now calculated to determine whether it lies on the curve as it should.

    3.4.1   Select a point on the curve about midway between two of the three points noted under 3.3 above, and read off the corresponding capacity, C_4 gpm, and pressure, P_4 psi.

    3.4.2

$$(E_w)_{C=C_4} = \text{constant} \quad \text{(see Section 2.2.42)}$$

Similar to Section 2.2.4.2.2 ,

$$(E_L)_{nom} = \boxed{B(2.54\ \ell)(30.5) \times 10^{-8}} \boxed{V_R - (V_L)_{nom}\ C_4/C_{nom}}$$

where all algebraic symbols have known values.

Now,

$$E_w - (i_1)_{C_4} R_w = (E_L)_{C_4} - (i_2)_{C_4} R_L$$

Solve for $(i_1)_{C_4}$ in terms of $(i_2)_{C_4}$

In addition,

$$E_w - (i_1)_{C_4} R_w = -\boxed{-(i_1)_{C_4} - (i_2)_{C_4}} \boxed{R_{co_{circ}} + R_{co_r} + R_{ci_{circ}} + R_{ci_r}}$$

substituting value of $(i_1)_{C_4}$ obtained just above, solve for

$$(i_2)_{C_4}$$

But,

$$(P_{developed})_{C_4} = (2n\,/A)(0.57 B(i_2)_{C_4}\,\ell \times 10^{-6} \quad \text{psi}$$

    (see Section 2.2.6.2)

$$(\text{Hydraulic loss})_{C_4} = (\text{hydraulic loss})_{nom}(C_4/C_{nom})^2$$

    (see Section 1.4)

Therefore,

$$(\text{Effective pressure})_{C_4} = (P_{developed\ C_4}) - (\text{hydraulic loss})_{C_4}$$

if the point C_4, P_{C_4} lies on the curve, this checks the curve.

75.

APPENDIX   B

Appendix B presents the equations for the generalized and simplified

design of an electrodynamic pump.   The following relations are derived:

(1)   Pump efficiency as a function of slip velocity

(2)   Pump inefficiency weight penalty

(3)   Theoretically ideal weight of the magnetic circuit

(4)   Pump weight/KW of mechanical output work

(5)   Weight penalty for consideration subcooling for a boiler feed pump

In addition, an example is calculated for the weight and weight

penalties of a boiler feed pump for a $2,000 KW_{(e)}$ space nuclear turbo-

electric power plant.   The comparative weights of the primary heat

exchange pump and condenser heat exchanger pump are also estima.ed,

and the total pump weight as a percentage of total power plant system

weight is calculated.

Symbols, units, and design assumptions are presented at the

beginning of the analysis.

SCHEMATIC ARRANGEMENT- ELECTRODYNAMIC PUMP

FIGURE 11
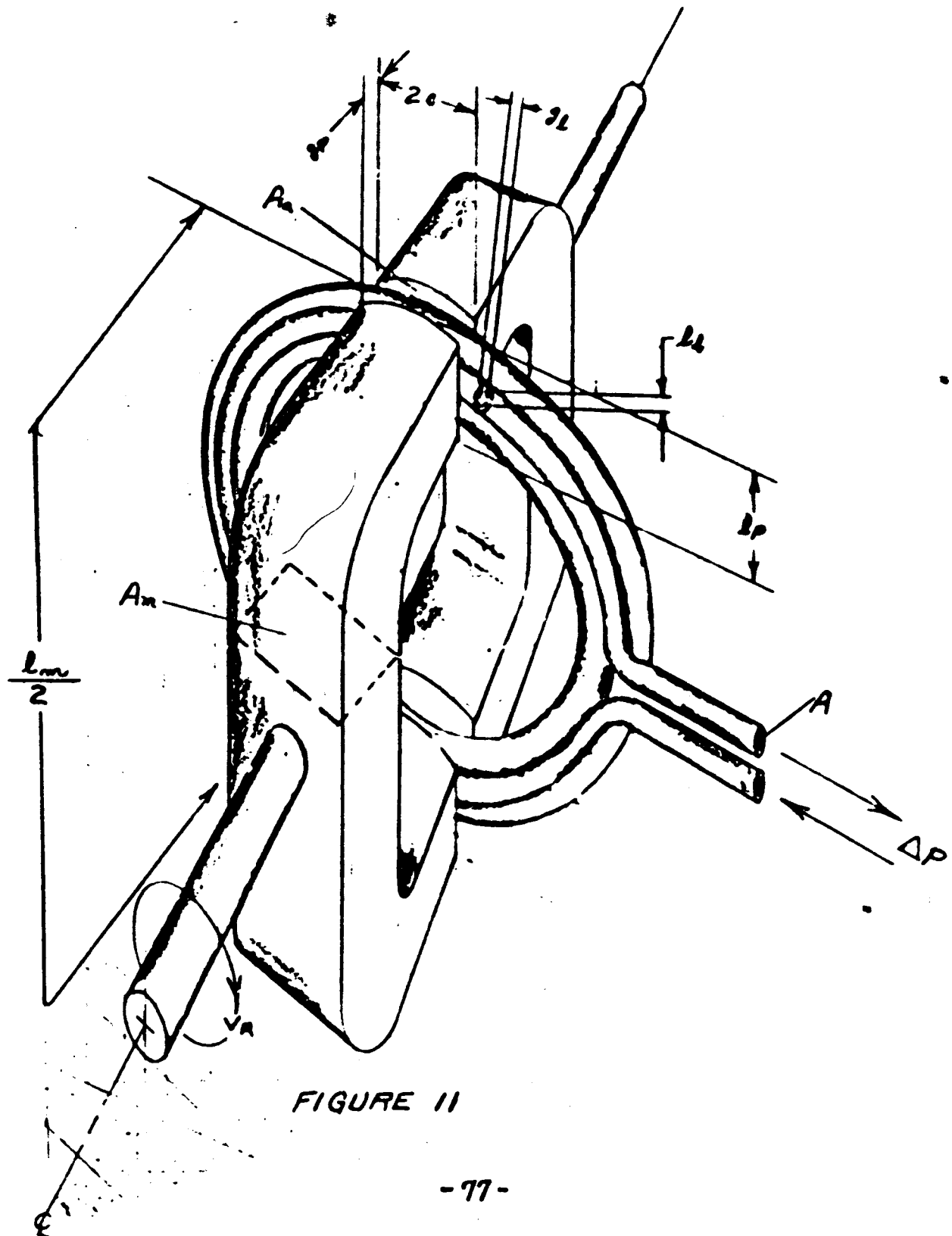
SYMBOL		DESCRIPTION	UNIT
A	$=$	Cross sectional area of the fluid passage $= g_\ell \ell_\ell$	in.^{2}
A_a	$=$	Cross sectional area of the air gap	in.^{2}
A_m	$=$	Cross sectional area of the magnetic material	in.^{2}
B	$=$	Magnetic flux density	gauss
B_m	$=$	Inductance of the magnetic material at demagnetizing force H_m	gauss
C	$=$	Fluid capacity	gal/min.
$2c$	$=$	Pole face dimension in circumferential direction	in.
d_m	$=$	Density of the magnetic circuit material	lbs/in.^{3}
E	$=$	Induced eddy voltage in the liquid	volts
F	$=$	Magnetofluid dynamic force developed on the fluid in the conduit	lbs.
g_ℓ	$=$	Thickness of liquid containing channel between poles	in.
g_p	$=$	Gap between pole faces	in.
H_m	$=$	Demagnetizing force of the air gap on the magnetic circuit	oersteds
I	$=$	Eddy current in one eddy	amps
ℓ_ℓ	$=$	Pump cell internal dimension in radial direction	in.
ℓ_m	$=$	Length of magnetic material	in.
ℓ_p	$=$	Pole face dimension in radial direction	in.
R	$=$	Total circuit resistance for one eddy	ohms
R_ℓ	$=$	Resistance of the liquid metal	ohms
V_ℓ	$=$	Liquid velocity	ft./sec.
V_m	$=$	Volume of magnetic material	in.^{3}

78.

SYMBOL		DESCRIPTION	UNIT
V_r	$=$	$V_\ell + V_s$ = Rotor velocity	ft/sec.
V_s	$=$	Slip velocity between liquid and poles	ft/sec.
W	$=$	Mechanical pumping work	Kw
W_m	$=$	Weight of magnetic material $= d_m V_m$	lbs.
ΔP	$=$	Developed pressure	$lbs/in.^2$
ϵ	$=$	Pump efficiency $= \dfrac{W}{W + \dfrac{E^2}{R}}$	dimensionless
μ_g	$=$	Permeability of the gap $= 3.19 \times 10^{-8}$	webers/amp-turn in.
σ_ℓ	$=$	Electrical resistivity of the liquid metal	ohm-in.
φ_g	$=$	Total magnetic flux in the gap	maxwells
φ_m	$=$	Total magnetic flux in the magnetic material	maxwells

DESIGN ASSUMPTIONS

1. The resistance of the fluid-containing walls is infinite in comparison with the resistance of the fluid.

    (Note: This assumption will never be entirely true as long as conducting material is used for fluid containment, but the wall loss can be made low by design.)

2. The resistance R_ℓ of the liquid metal is 70% of the total circuit resistance, R.

3. The hydraulic loss of the fluid flowing through the pump cell is small and will be assumed to be 10% of the developed pressure in this analysis.

4. The following geometry will be arbitrarily set:

    $$g_\ell = 0.6\, g_p$$

    $$\ell_\ell = \ell_p = 10\, g_p \quad \text{for which} \quad \varphi_m = 2\, \varphi_g$$

    $$V_\ell = 50 \text{ ft sec.}$$

    $$R = \frac{R_\ell}{0.7} \quad \frac{\sigma_\ell \dfrac{\ell_\ell}{g_\ell^2 c}}{0.7} = \frac{\sigma_\ell \dfrac{10\ g_p}{0.6\ g_p^2 c}}{0.7} = \frac{11.9\ \sigma_\ell}{c}$$

5. The following fluid properties and magnetic properties will be assumed:

    $$B = B_m = 10,700 \text{ gauss} = \text{inductance of Alnico 7}$$
    for maximum energy product for which

    $$H_m = 650 \text{ oersteds} \qquad d_m = 0.265 \text{ lbs in.}^3$$

    $$\sigma_\ell = 17 \times 10^{-6} \text{ ohm-in.} \text{ (about equivalent to the resistivity}$$
    of sodium (or lithium) at 1400° F.

6. C, the fluid capacity in gpm, and ΔP, the developed pressure in lbs. in.^{2}, will be fixed by the requirements of the power plant system. In the analysis they will be generalized to give the answers for the pump weight and weight penalty for inefficiency per Kw of mechanical output work.

7. g_p, C and V_s are held as the independent variables, and it is found that g_p and C drop out of the expression when the pump weight is given for 1 Kw of mechanical output work, leaving only V_s as an independent variable. Likewise, the same thing occurs in the case of the efficiency ϵ, which becomes a function of only the slip velocity, V_s.

L. The capacity equation

The fluid capacity is given by

$$C = 3.12 \, g_\ell \, \ell_\ell \, V_\ell \quad \text{gpm}$$

Substituting design assumption values for $g_\ell \, \ell_\ell \, V_\ell$ gives

$$C = 936 \, g_p^2 \quad \text{gpm}$$

II. The developed pressure equation

$$\Delta P = \frac{F}{A} \quad \text{lbs in.}^2$$

where

$$A = g_\ell \ell_\ell \quad \text{in.}^2$$

$$F = .57 \, B \, I \, \ell_\ell \times 10^{-6} \quad \text{lbs.}$$

$$I = \frac{E}{k}$$

$$E = 7.76 \, B \, \ell_\ell \, V_s \times 10^{-7} \quad \text{volts.}$$

Substituting $B = 10,700$ gauss $\quad \ell_\ell = \ell_p = 10 \, g_p$

$$E = 0.083 \, V_s \, g_p \ ;$$

substituting $\dfrac{11.9 \, \sigma_\ell}{c} = R \qquad \sigma_\ell = 17 \times 10^{-6}$ ohm-in.

$$I = 4.12 \, V_s \, g_p \, c \times 10^2 \ ;$$

substituting in the equation for F

$$F = 25.1 \, V_s \, g_p^2 \, c \ ; \text{and}$$

substituting for g_ℓ, and ℓ_ℓ in A,

then, $\quad \Delta P = 4.18 \, V_s \, c$

III. The mechanical work equation

$$W = 4.35 \, \Delta P \, C \times 10^{-4} \quad Kw$$

Substituting for ΔP and C,

$$W = 1.7 \, V_s \, g_p^2 \, c$$

IV. $I^2 R$ loss equation

$$I^2 R = \frac{E^2}{R}$$

Substituting for E and R gives

$$I^2 R = \frac{(.083)^2 \, V_s^2 \, g_p^2 \, c}{2.03 \times 10^{-4}} = \underline{34 \, V_s^2 \, g_p^2 \, c} \quad watts =$$

$$\underline{3.4 \, V_s^2 \, g_p^2 \, c \times 10^{-2} \quad Kw}$$

Loss per Kw of mechanical work =

$$I^2 R = \frac{3.4 \, V_s^2 \, g_p^2 \, c \times 10^{-2}}{1.7 \, V_s \, g_p^2 \, c} = \underline{2.0 \, V_s \times 10^{-2} \, Kw}$$

V. Pump efficiency,

$$\epsilon = \frac{W}{W + I^2 R} = \frac{1.7 \, V_s \, g_p^2 \, c}{1.7 \, V_s g_p^2 \, c + 3.4 \, V_s^2 \, g_p^2 \, c \times 10^{-2}}$$

$$\epsilon = \frac{1.7}{1.7 + .034 \, V_s}$$

This function is plotted against V_s in ABC - 153.

SLIP VELOCITY INFLUENCE ON PUMP WEIGHT

ABC-153
5-8-62
L8
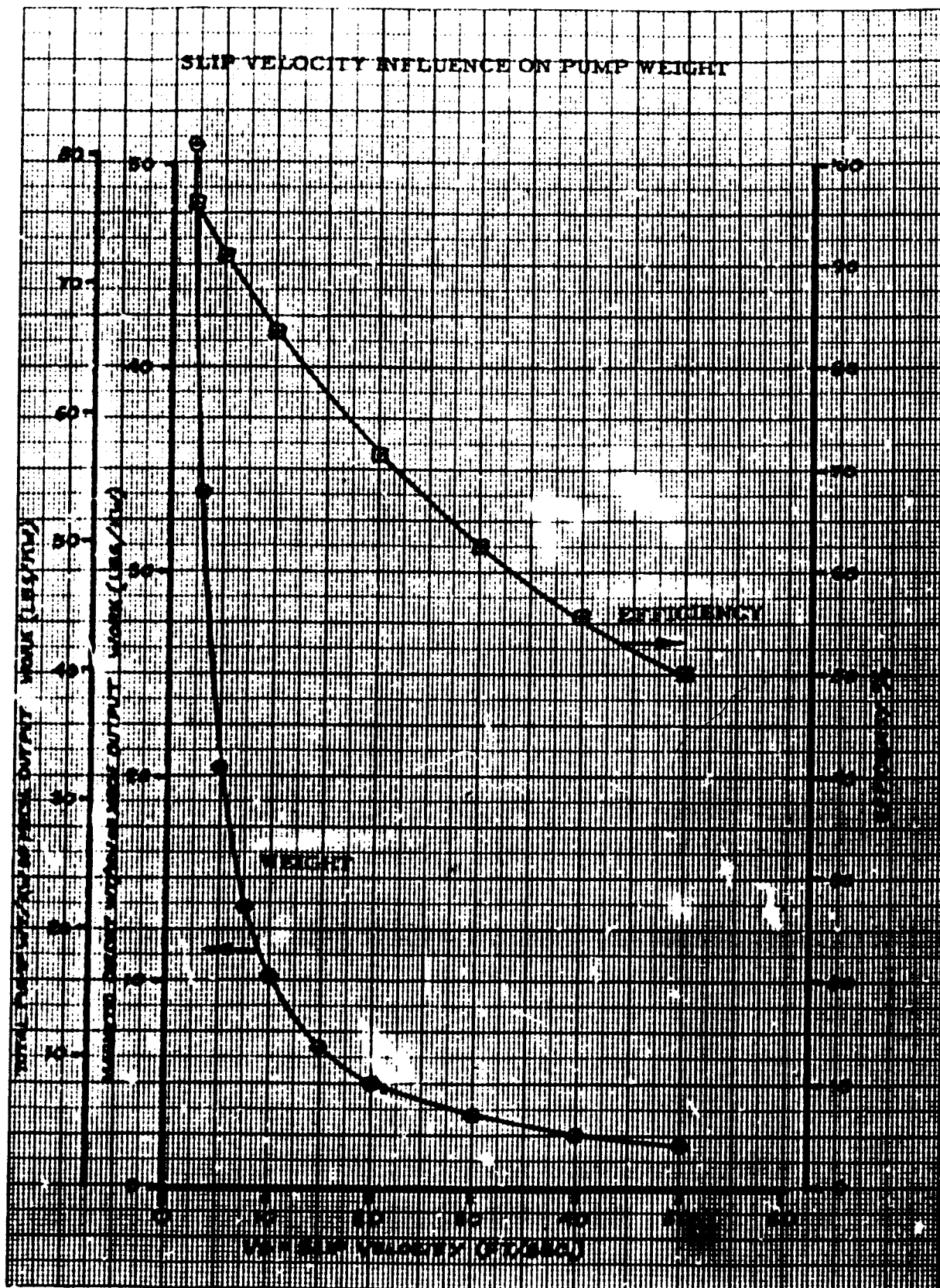
VI. Pump Inefficiency Weight Penalty

For any type of pump with effeciency ϵ in a power plant system weighing X lbs/Kw of electrical output, the pump must assume a weight penalty equal to the extra system weight that must be added to generate the power dissipated by the pump as inefficiency.

For 1 Kw of useful pump mechanical output work, the pump inefficiency weight penalty is given by

$$\left(\frac{1}{\epsilon} - 1\right) X, \quad \text{where } X = \text{weight/Kw of power plant}$$

This pump inefficiency weight penalty is plotted against efficiency for power plant weights /Kw of 1, 5, 10, 20, 40, 50, and 60 lbs. in drawing. D - 11413.

VII. Magnetic Circuit Weight - Pump Weight per Kw of Mechanical Output Work.

The volume of magnetic material is

$$V_m = \ell_m \, A_m$$

Then for a circuit in which half of the flux in the magnetic circuit is useful flux, or $\varphi_m = 2\varphi_g$,

$$\ell_m A_m = \frac{3.2 \; B^2 A_a \; g_p}{\mu_g H_m B_m} \times 10^{-8}$$

Substituting for μ_g , H_m , B_m , and $A_a = 20 \, c \, g_p$. and B.

$$V_m = \ell_m A_m = 658 \, c \, g_p^{\,2}$$

85.

PUMP EFFICIENCY VS WEIGHT PENALTY ~LBS/KW ~ FOR VARIOUS POWER PLANT SYSTEM WEIGHT (TOTAL) PER KW OF EL

FOR PUMP EFFICIENCY FROM 6% TO 100% AND FOR SYSTEM TOTAL WEIGHT/KW OF POWER GENE 1 LB/KW, 5 LB/KW, 10 LB/KW, 20 LB/KW, 40 LB/KW, 80 LB/KW

PUMP EFFICIENCY %

1 LB/KW

5 LB/KW

10 LB/KW

WEIGHT PENALTY   LBS.
MECH. PUMP WORK   KW

WGHT PENALTY in LBS./KW of MECH. PUMP WORK

ITEM WEIGHT (TOTAL) PER KW OF ELECTRICAL OUTPUT

FROM 10% TO 100%

WEIGHT/KW OF POWER GENERATED OF
KW, 20 LB/KW, 40 LB/KW, 50 LB/KW, 60 LB/KW

REF:
$$LOSS = \left(\frac{1}{EFF} - 1\right) W_b \ (LB/KW)$$

EX:
FOR EFF = 30%, W_b = 60 LB/KW

$$LOSS = \left(\frac{1}{.3} - 1\right) 60 \ LB/KW$$

$$LOSS = 139 \ LB/KW$$

PUMP EFFICIENCY %

10 LB/KW   20 LB/KW   40 LB/KW   50 LB/KW   60 LB/KW

LBS./KW

100
90
80
70
60
50
40
30
20
10

20 40 50 60 70 80 90 100   200   300   400   500 600

D 1413

PUMP EFFICIENCY
VS WEIGHT PENALTY

D 1413

The weight of the magnetic material is

$$W_m = d_m V_m \ ,$$

Substituting for d_m and V_m ,

$$W_m = 174 \ c \ g_p^2 \ \text{lbs.}$$

The magnetic circuit weight/Kw of mechanical output is given by

$$\frac{W_m}{W} = \frac{174 \ c \ g_p^2}{1.7 \ V_s g_p^2 c} = \frac{102.5}{V_s}$$

If the magnetic circuit represents 70 % of the pump weight, and the hydraulic loss represents 10% of the developed head, then the total pump weight/ Kw of mechanical output work will be

$$= \frac{102.5}{V_s \ x7x .9} = \frac{162.5}{V_s} \ \text{lbs/Kw}$$

The magnetic circuit weight/Kw and the total pump weight/Kw are plotted against the slip velocity in ABC - 153.

VIII. **Weight Penalty for Condensate Subcooling in Boiler Feed Pump:**

In any boiler feed pump it will be necessary to subcool the liquid condensate sufficiently to prevent cavitation at the inlet. Subcooling will require additional heat removal by the radiator and at a lower temperature than the vapor condensation temperature.

As an example of this weight penalty a vapor turbine cycle employing sodium as the working fluid with an overall cycle efficiency of .15 was selected. The weight penalty per Kw of power plant electrical output is plotted against $^{\circ}F$ of subcooling

87.

for radiator weight/ft.^{2} divided by the radiator surface emmissivity ranging from 1 to 10 lbs/ft.^{2} (see Drawing D - 11414)

IX. Pump Design Example

As an example of the pump design determined by the above 7 sections, the boiler feed pump for a 2,000 $Kw_{(c)}$ power plant using sodium as the working fluid and having an overall cycle efficiency of .15 and total power plant weight of 10 lbs/Kw was selected. The radiator weight was assumed equal to 1/3 of the total system weight or equal to 4.75 lbs/ft.^{2} of surface at an emissivity of 1. The condenser radiator was assumed to operate at 1400° F.

The boiler feed pump capacity would be 65 gpm with a total developed pressure of 300 psi

The mechanical work will then be

$$W = 4.35 \times 65 \times 300 \times 10^{-4} \text{ Kw} = 8.5 \text{ Kw}$$

The system weight is 10 lbs/Kw. Using ABC - 153 and D - 11413, it is found that for V_s = 30 ft/second, the efficiency = 62.5 %, and the magnetic circuit weight is 3.39 lbs/Kw, and the total pump weight is 5.38 lbs/Kw. From D - 11413 the weight penalty is 6 lbs/Kw for an efficiency of 62.5 % and a system weight of 10 lbs/Kw. Then the total pump weight (exclusive of drive) is 8.5 x 5.38 = 45.7 lbs. , and the efficiency penalty is 8.5 x 6 = 51 lbs.

With an entrance inlet velocity of 5 ft/second, the theoretical degree of subcooling required would be about 10°F. (6.93 ft/second critical velocity). From D - 11414 the condensate subcooling

88.

LBS/KW OF SYSTEM OUTPUT POWER (×10⁶)

10 LBS/FT²

8

400
375
350
325
300
275
250
225
200
175
150
125
100
75
50
25
0

0	10	20	30	40	50	60	70	80	90	100	110	120
	3.64	7.28	10.92	14.57	18.4	21.85	25.5	29.1	32.8	36.4	40.1	43.7

REF 1

$$KWE = 2000\ KW$$

$$LBS/KW = \frac{A(FT^2) \times LBS/FT^2}{2000\ KW}$$

$$= \frac{18.4\ FT^2 \times 10\ LBS/FT^2}{2000\ KW} =$$

$$91.2 \times 10^{-3}\ LBS/KW$$

10 LBS/FT²

8 LBS/FT²

5 LBS/FT²

2 LBS/FT²

1 LB/FT²

LBS/KW OF SYSTEM OUTPUT POWER (×10⁻³)
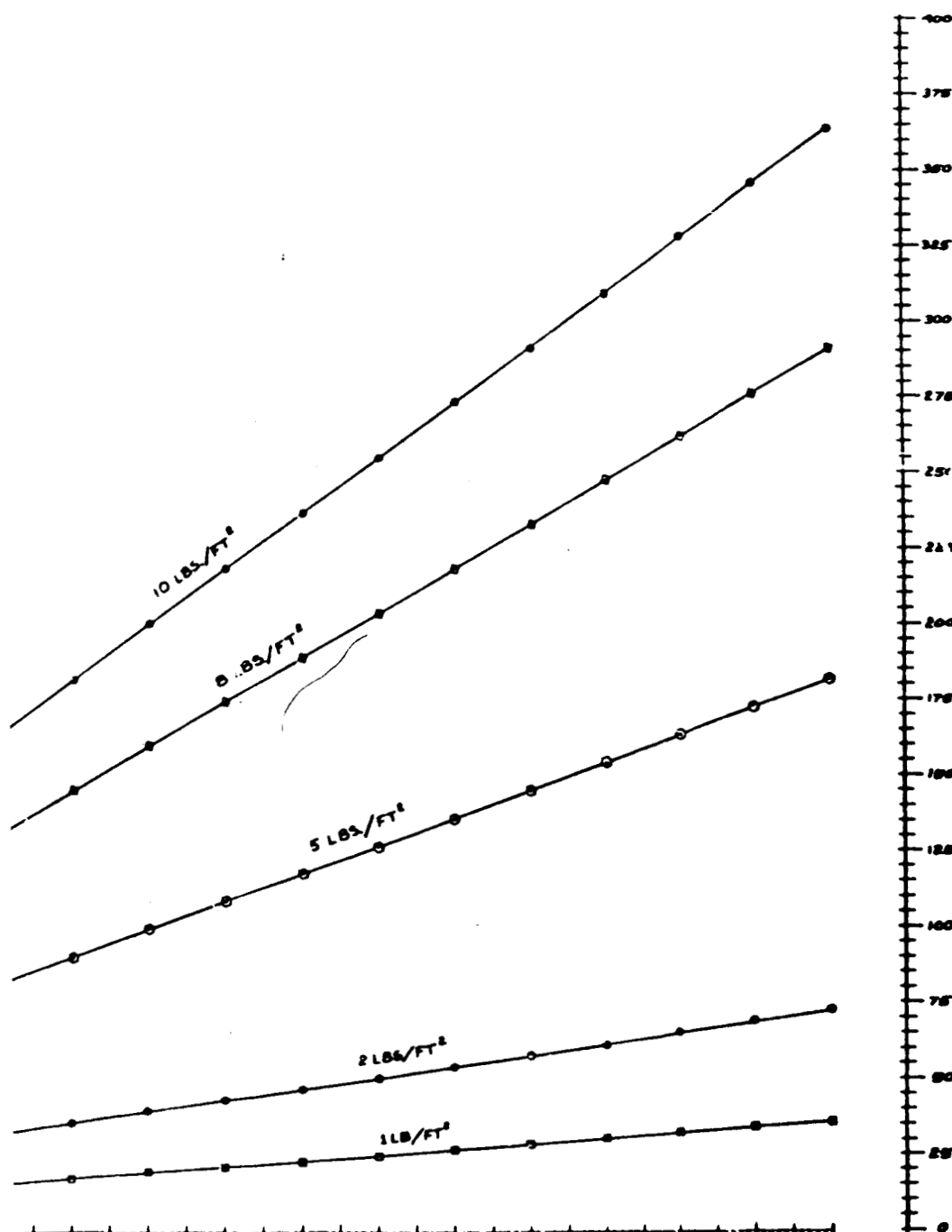
| | | | | | | | | | | | | | | | | | |
| 400 | 375 | 350 | 325 | 300 | 275 | 250 | 225 | 200 | 175 | 150 | 125 | 100 | 75 | 50 | 25 | 0 |

100	110	120	130	140	150	160	170	180	190	200	— CHANGE IN TEMP BELOW BODY BASE TEMP.	ΔT
36.4	40.1	43.7	47.3	51.0	54.7	58.3	61.0	64.6	68.2	72.8	— RADIATOR AREA REQUIRED FOR Δ TEMP. (FT²)	A

| | D | 11414 | |

weight penalty can be read off as 8×10^{-3} lbs/Kw of system output power for a value of radiator weight/ft^2 divided by radiator surface emissivity = 4.75. For a 2,000 Kw$_{(e)}$ system, the absolute weight penalty will be

$$2,000 \times 8 \times 10^{-3} = \underline{16 \text{ lbs.}}$$

Then the total pump weight and pump weight penalties will be 112.7 lbs. It is likely that in this system there would be two additional pumps: i.e. , primary heat exchange and condenser heat exchange pumps. While the capacity would be about 10 times that of the boiler feed pump for each of these applications, the developed pressure would be only about 10% of that required for the feed pump, so that the total mechanical work for each of these pumps would be equivalent, and hence the total weight and penalties would be about the same.

The combined weight and penalties for the three pumps in the system would then be $3 \times 112.7 = 338.1$ lbs. or $\frac{338.1}{20,000} = 1.69\%$ of the total system weight.

APPENDIX III

Calculated and Experimental Performance Data for Several

Electrodynamic Pumps Studied.

THEORETICAL PERFORMANCE CURVES   PUMP #1200

EFFECT OF CHANGING THE FIELD CURRENT

EFFECT OF CHANGING THE TEMPERATURE OF THE K

LIQUID METAL   POTASSIUM, K

ROTOR SPEED   1200 RPM

FIELD CURRENT   As Marked

FLOW RATE POTASSIUM, GPM

THEORETICAL PERFORMANCE CURVES   PUMP #799

EFFECT OF CHANGING THE WALL THICKNESS
OF THE LIQUID METAL CONTAINING PORTION
OF THE CELL
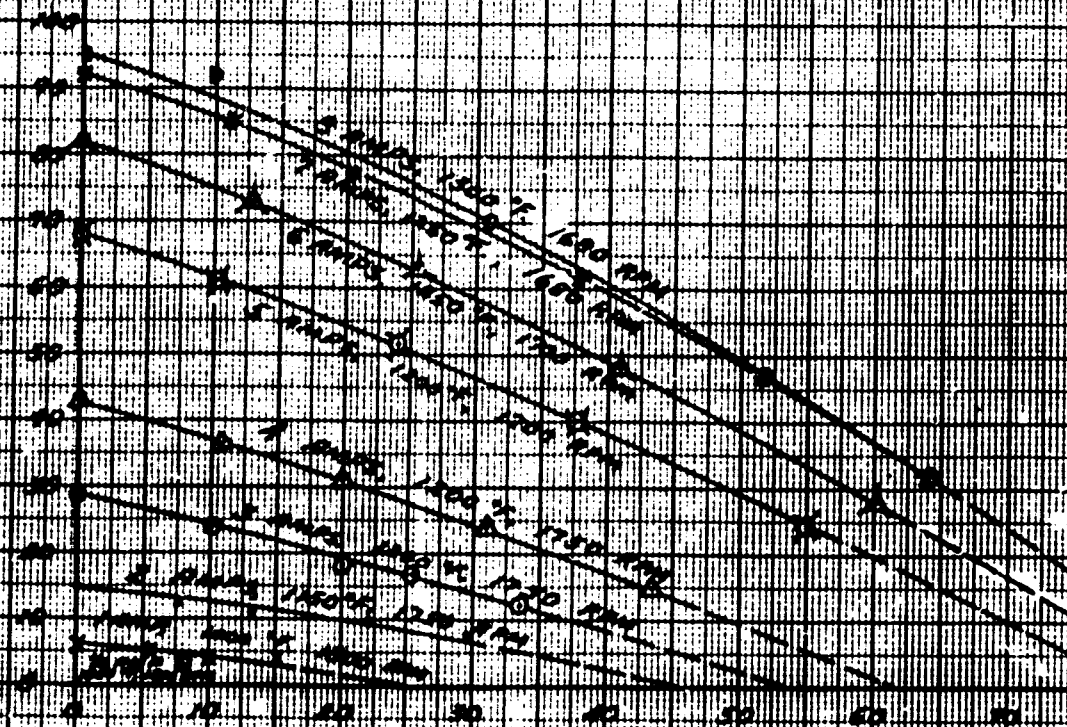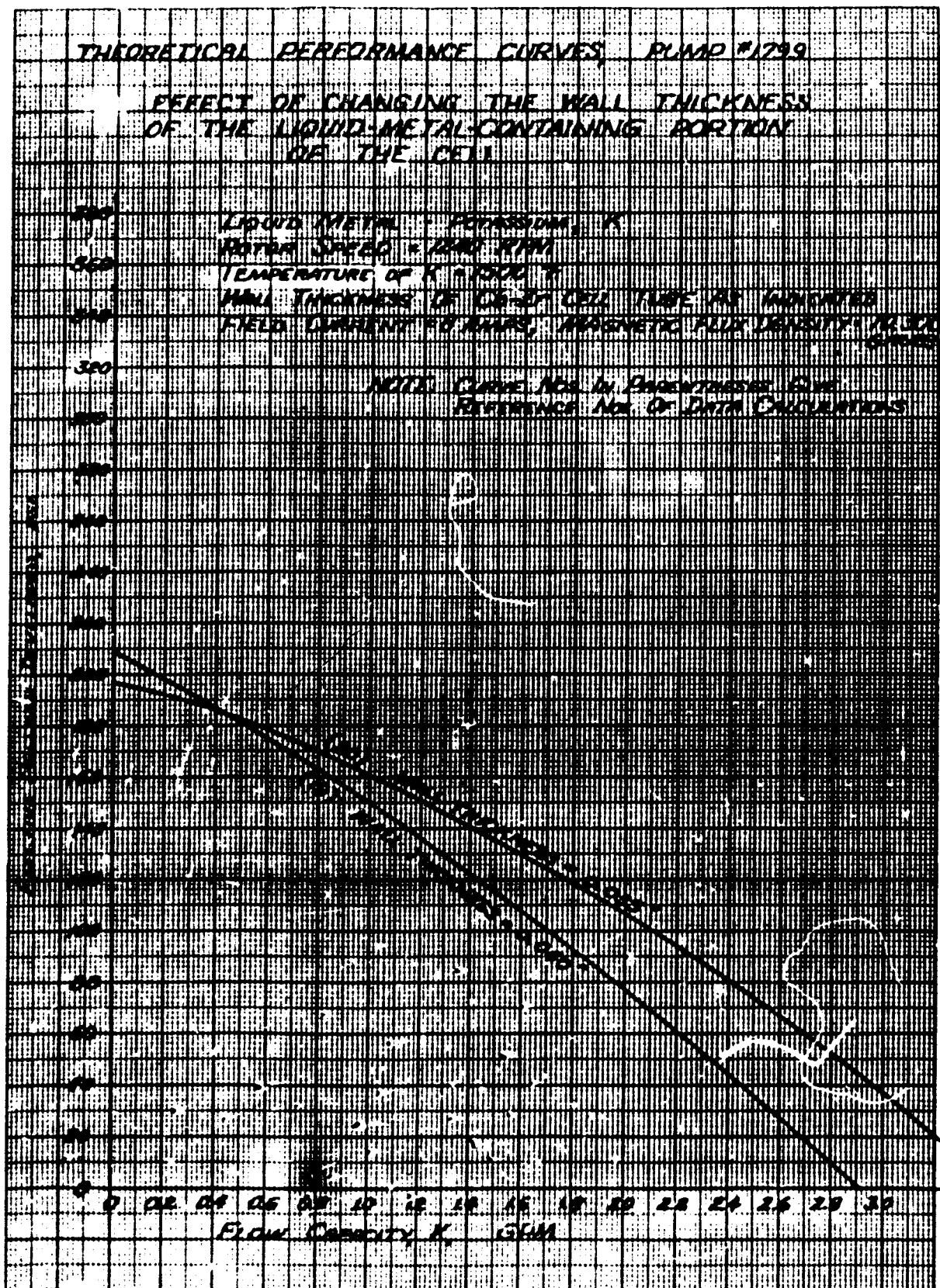
THEORETICAL

ELECTRICAL LOSSES IN PUMP CELL AS A FUNCTION OF

FIELD CURRENT AND OF POTASSIUM FLOW RATE

For Pump Cell #1799 Having A Cb-1%-Zr Liquid-Metal-
Containing Tube With Copper Conductor Rings,
The Whole Being Housed In An Inconel Sheath.

Liquid Metal - Potassium (K)
Rotor Speed - 1240 RPM
Temperature Of K - 1500°F
Wall Thickness Of Cb-1%-Zr Tube - 0.035
Field Current And Magnetic Field As Indicated

FIELD CURRENT AMPS	MAGNETIC FLUX DENSITY GAUSS	POTASSIUM FLOW RATE GPM	I²R LOSSES IN CELL WALL	I²R LOSSES IN HOUSING	TOTAL LOSSES APPROXIMATELY HP
0	16,300	0	1020	595	3.2
8	16,300	1	1673	595	3.0
5	8,100	0	1120	3760	4.0
5	8,100	1	1630	3760	2.9

Pump # 17005, 6" Pole Circle Diameter
Actual Test Runs: K @ approx. 930 F
  Rotor Speed = 2600 rpm
  Cell Type: 347 s/s, 2:1 ellipse formed from
  3/8" O. D. x .035" wall tubing
  Air Gap: 0.406"
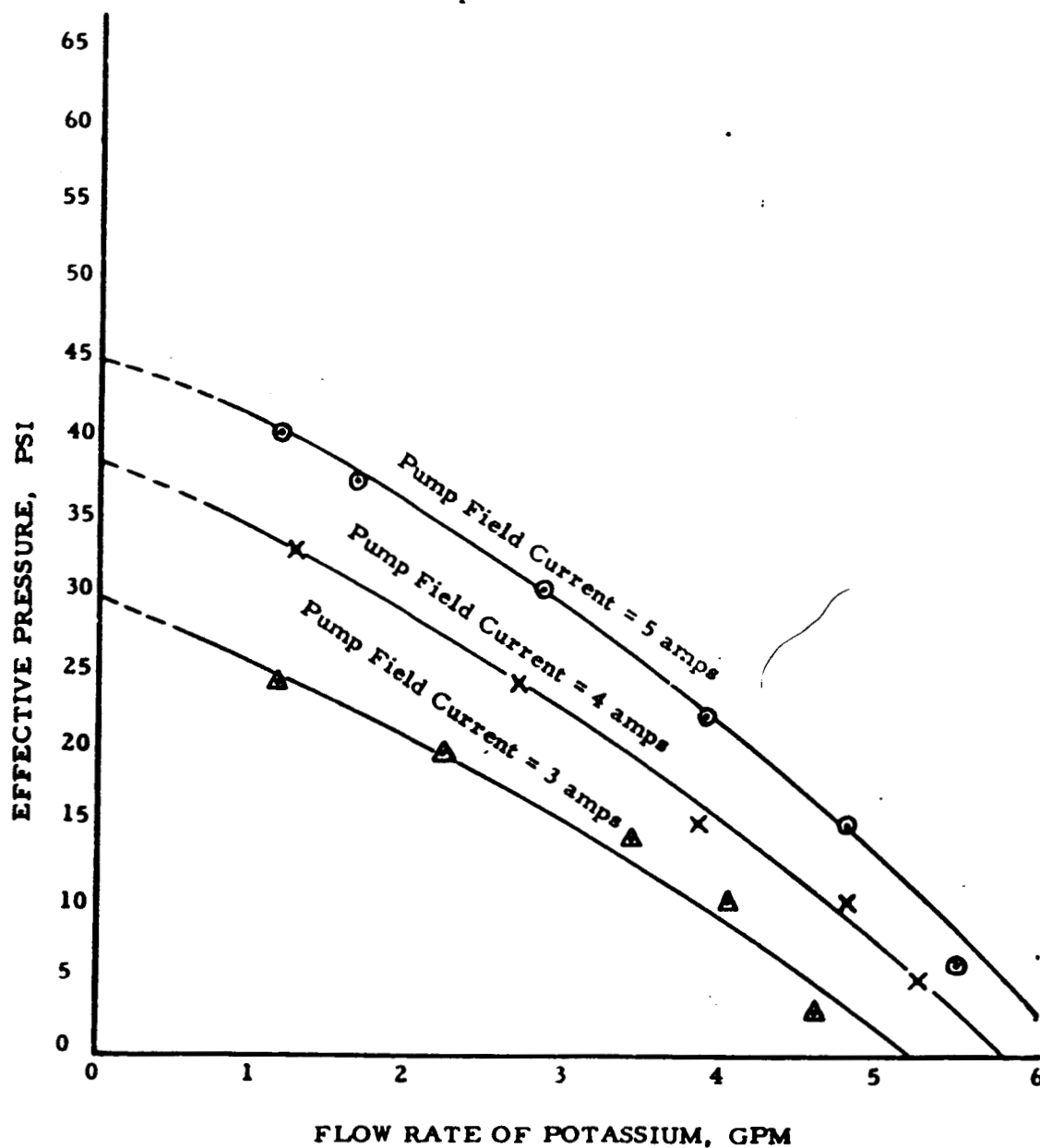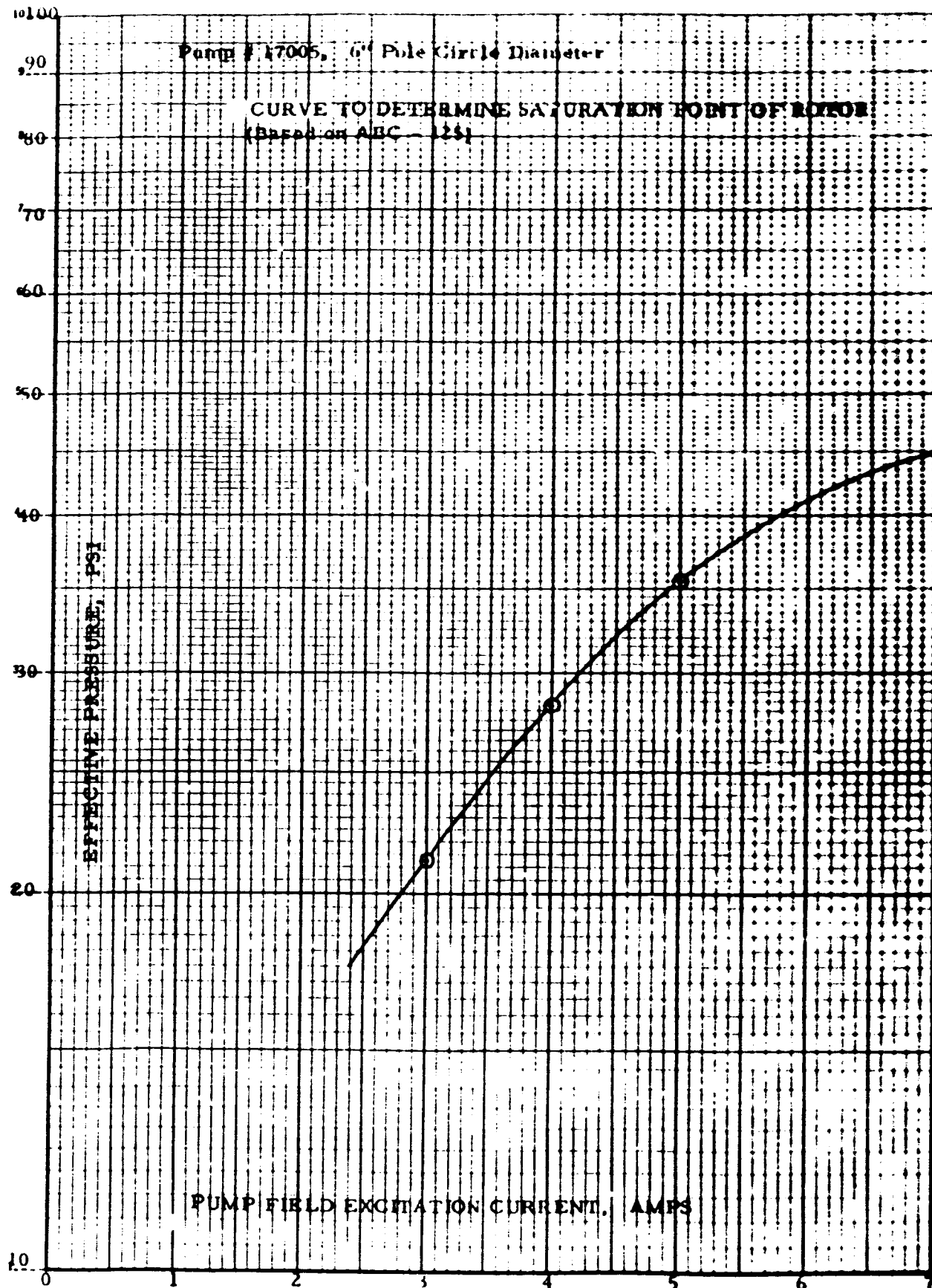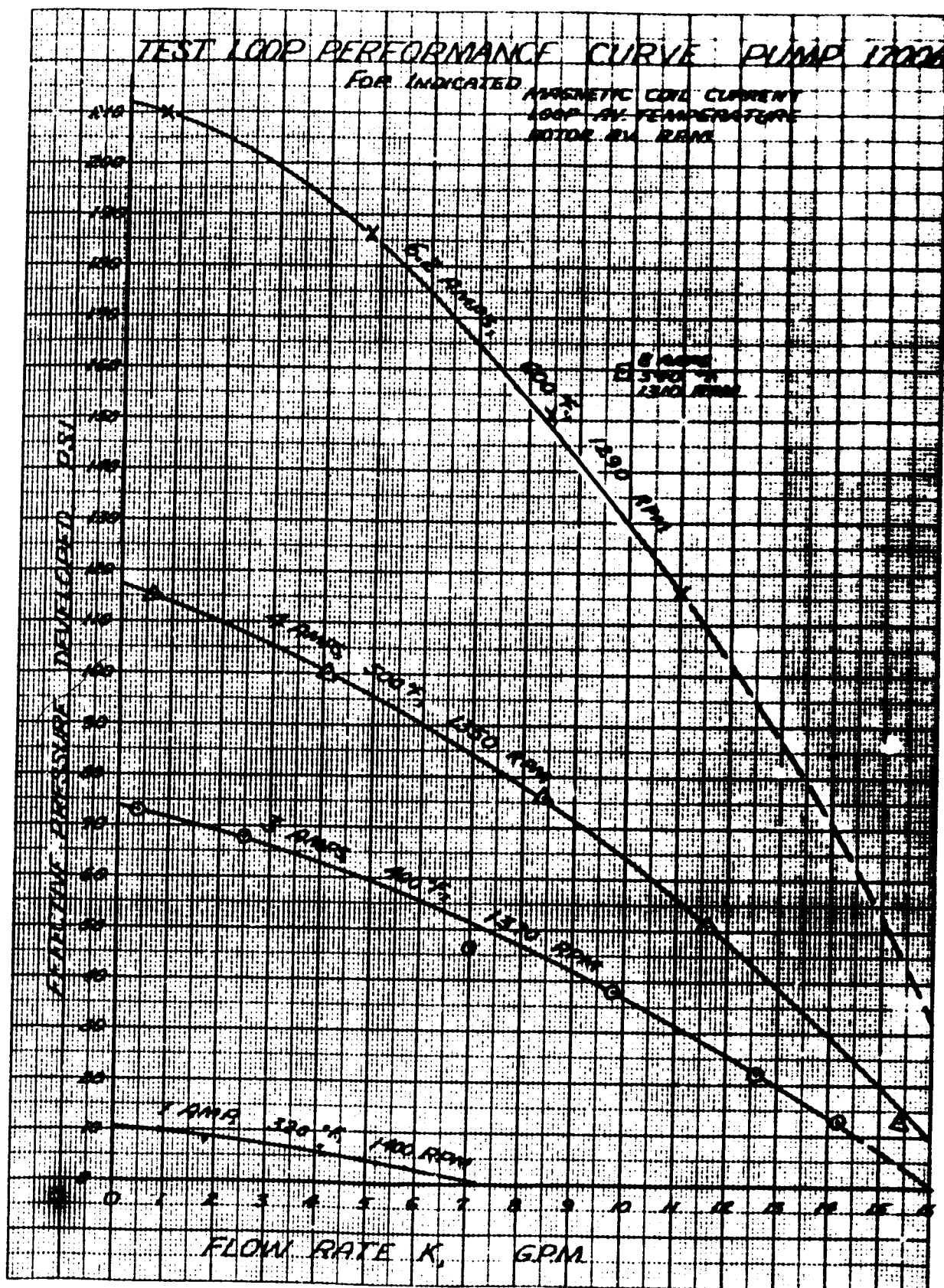
FLOW RATE OF POTASSIUM, GPM

(Y-axis: EFFECTIVE PRESSURE, PSI)

Pump Field Current = 5 amps
Pump Field Current = 4 amps
Pump Field Current = 3 amps

Pump # 47005,   6" Pole Circle Diameter

CURVE TO DETERMINE SATURATION POINT OF ROTOR
(Based on ABC - 125)

EFFECTIVE PRESSURE, PSI

PUMP FIELD EXCITATION CURRENT, AMPS

JMF 14 XI 61

TEST LOOP PERFORMANCE CURVE PUMP 17001

ABC-129.3

TEST RUN PERFORMANCE CURVES FOR ELECTRODYNAMIC PUMP

Model P12H0.15-1-387.5   517012/1

Rotor Speed	920 rpm
Pole Gap	0.5625
Pole Height	0.427

JMF 6162

-99-

TEST RUN PERFORMANCE CURVES FOR ELECTRODYNAMIC PUMP

Model P12H0.15-143B7.5   #17012/2

Rotor Speed 920 rpm
Pole Gap 0.562"
Pole Height: 0.427"
Temperatures taken on discharge side of pump cell
DC currents also thru pump field windings

Flow Rate, Potassium, GPM
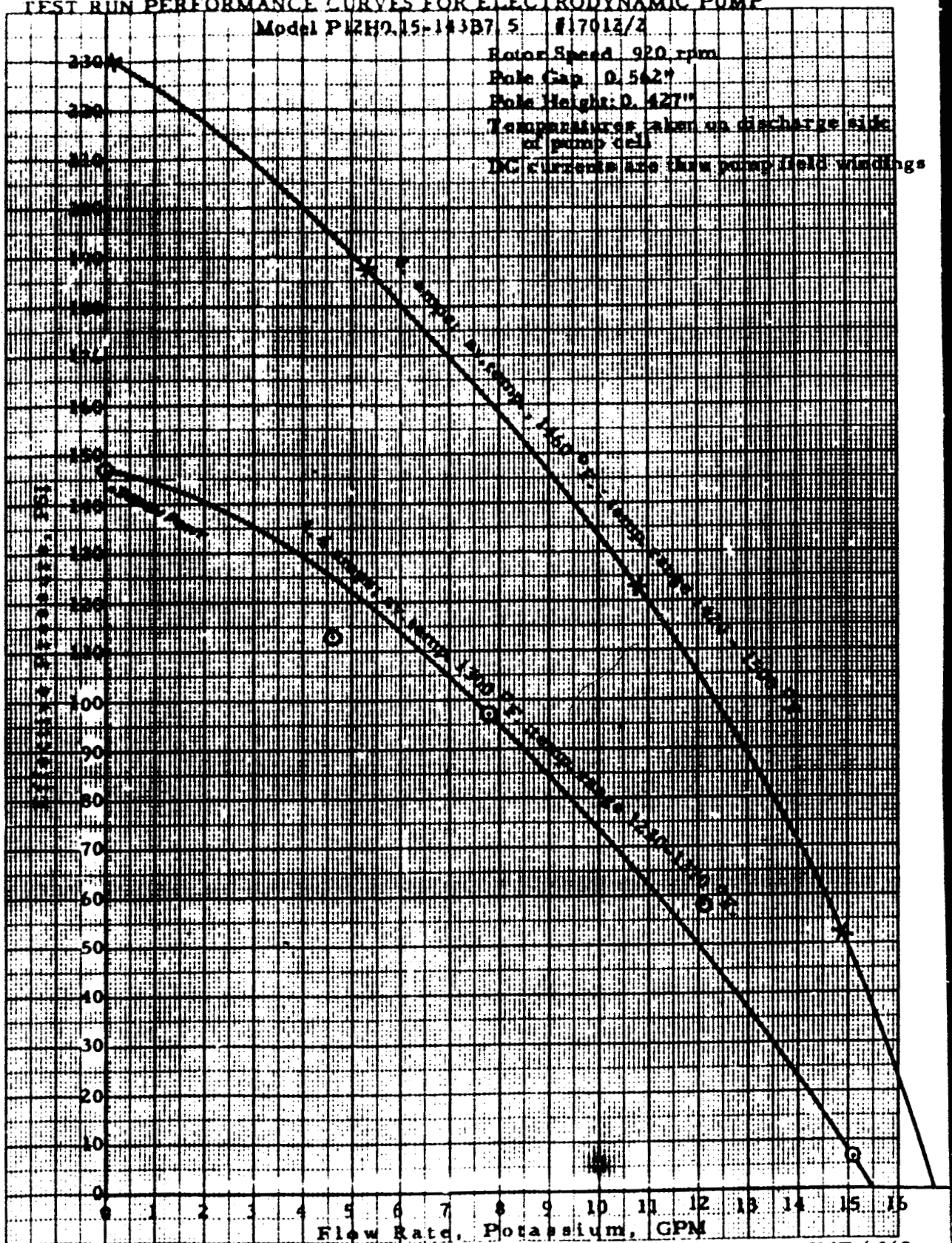
JMF 6162

#1012 Pumps #1, 2

ACTUAL PERFORMANCE CURVES FOR EDP Model P12H0.15-143B7.5: Two Pumps

Rotor Speed   920 rpm
Pole Gap   0.562"
Pole Ht.   0.427"
Temperatures:

Flow Rate — GPM

JMF 6162

ABC-139
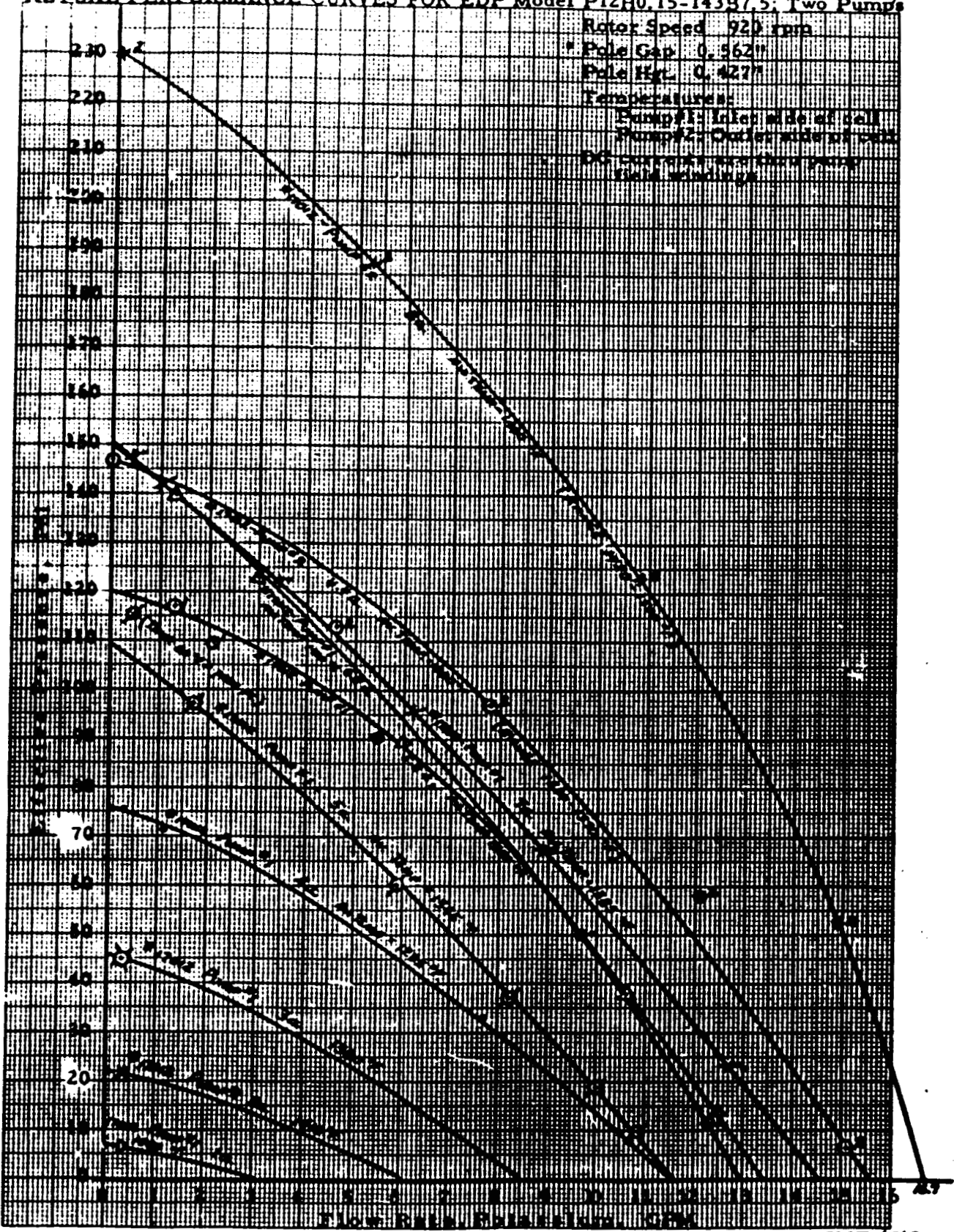
-102-
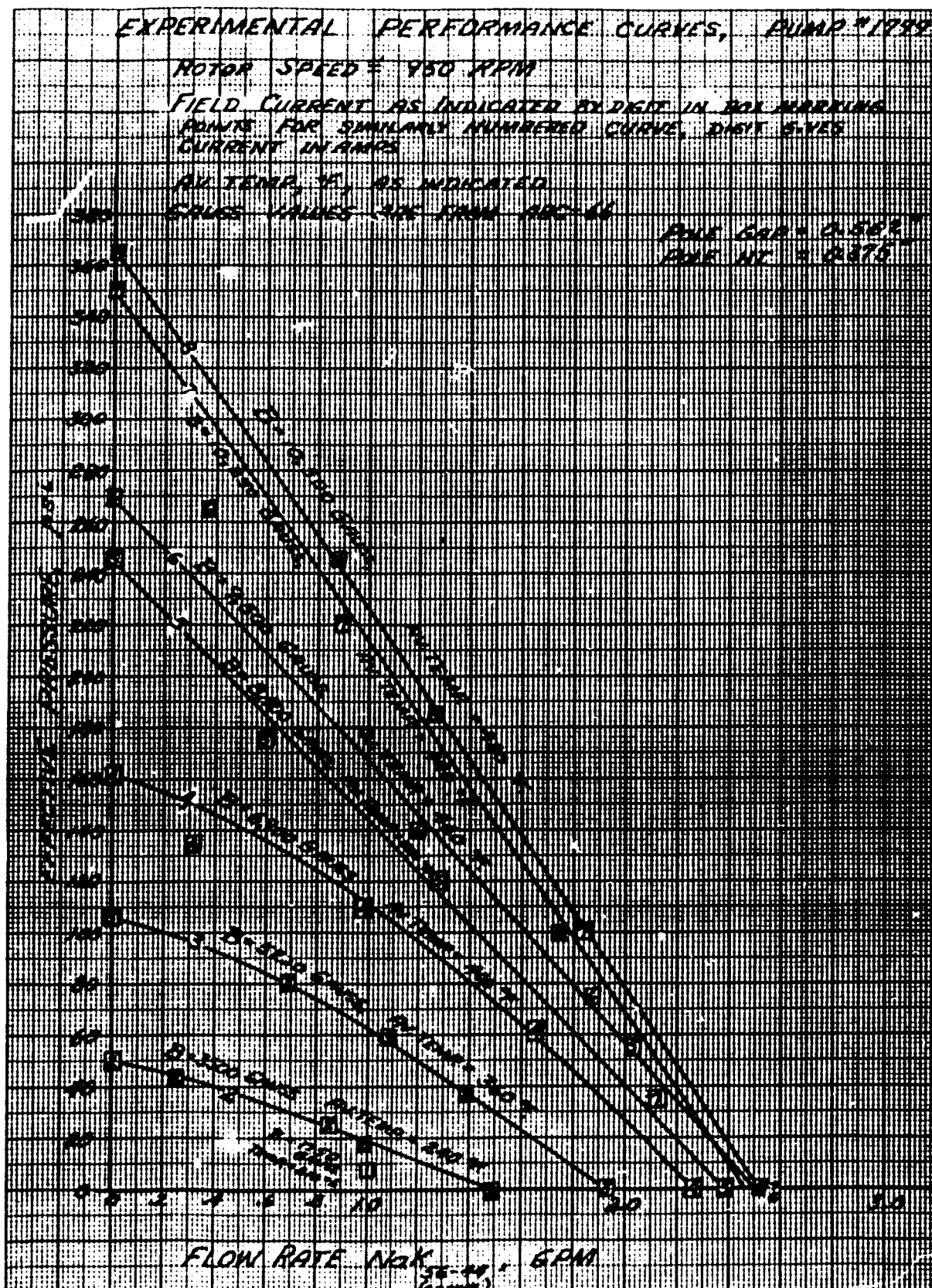
EXPERIMENTAL PERFORMANCE CURVES, PUMP #1999

ROTOR SPEED ~ 1250 RPM

Av. TEMP AND FIELD CURRENT AS INDICATED

POLE GAP = 0.562"
POLE WT. = 0.375"

GAUSS VALUES ARE FROM ABC-66

NOTE ON EFFECT OF TEMPERATURE:
TEMP FACTOR GIVES FLOW RATE
4% LOWER AT 500°F THAN AT 280°F

FLOW RATE NaK, GPM

ABC-143

THEORETICAL PERFORMANCE CURVES, PUMP #1799

EFFECT OF CHANGING ROTOR SPEED

LIQUID METAL — NaK-44

FIELD CURRENT — 3 amps (B=5150 Gauss)

LEGEND

CURVE (A) — ROTOR SPEED = 950 RPM

CURVE (B) — ROTOR SPEED = 1250 RPM

CURVE (C) — EXPERIMENTAL

FLOW RATE NaK-44, GPM

EFFECTIVE LIQUID DISCHARGE PRESSURE, PSI

- 104 -

THEORETICAL PERFORMANCE CURVES, PUMP #1799

EFFECT OF ARBITRARILY CHANGING THE GENERATED POTENTIAL VALUE CALCULATED FOR A ROTOR SPEED OF 950 RPM (The change in E is equivalent to a change in rotor speed)

LIQUID METAL   NaK, 56-44
FIELD CURRENT   3 AMPS   (B= 5150 GAUSS)
TEMPERATURE OF NaK   360 °F
ELECTRICAL POTENTIAL GENERATED IN WALL OF LIQUID METAL CONTAINING TUBE SYMBOLIZED BY E_w, volts/amp

MAGNETIC POLE GAP G = .562"
"   "   W = .375"

LEGEND
CURVE (5), same as (5) on ABC-141, E_w=0.055 volts
CURVE (6), E_w arbitrarily taken as 0.065 volts
CURVE ③, EXPERIMENTAL, same rotor speed (950 RPM), hence same E_w as for Curve (5).
CURVE ③ TAKEN FROM GRAPH ABOVE

EFFECTIVE PRESSURE, PSI. (CORRECTED AS NOTED UNDER [3], GRAPH ABC-140)

FLOW RATE NaK 56-44, GPM

THEORETICAL PERFORMANCE CURVES, PUMP # 1799

EFFECT OF CHANGING COMPOSITION, HENCE ELECTRICAL
RESISTIVITY, OF THE NaK BEING PUMPED

LIQUID METAL NaK, approx 56-44

ROTOR SPEED = 930 RPM
FIELD CURRENT = 3 AMPS (B = 3150 GAUSS)

TEMPERATURE OF NaK = 360 °F
ELECTRICAL RESISTIVITY SYMBOLIZED BY ρ

MAGNETIC POLE GAP = 0.375"

CURVE (1) THEORETICAL FOR ρ_{NaK}
CURVE (2) THEORETICAL FOR ρ_{NaK}

CURVE (3) EXPERIMENTAL FOR CONDITIONS NOTED
UNDER HEADING OF THIS GRAPH
(SEE ALSO GRAPH AHL-139)

DISCHARGE PRESSURE WAS MEASURED DOWN
STREAM FROM 25" LENGTH OF 1/4" DIA. CV. WALL
TUBING THRU FLOWMETER. HYDRAULIC LOSS
FOR THIS SECTION OF TUBING IS GIVEN ON
GRAPH AHL-116
THE HYDRAULIC LOSSES THRU THIS PIPE
MUST BE DEDUCTED FROM THEORETICAL
CURVES TO OBTAIN EQUIVALENCY TO EXPTL
CURVES. THEREFOR,

CURVE (1)c CURVE (1) CORRECTED FOR FLOWMETER AND
HYDRAULIC LOSS
CURVE (2)c CURVE (2) CORRECTED FOR FLOWMETER AND
HYDRAULIC LOSS

FLOW RATE NaK 56-44, GPM
approx

THEORETICAL PERFORMANCE CURVES, PUMP #1799

EFFECT OF CHANGING THE ASSUMED ELECTRICAL
RESISTIVITY VALUE OF Cb Zr, for which no data
at elevated temperatures was available

LIQUID METAL - NaK 1131
ROTOR SPEED - 550 RPM
FIELD CURRENT - 5 AMPS (2.5750 GAUSS)
TEMPERATURE OF NaK 560 °
ELECTRICAL RESISTIVITY SYMBOLIZED BY ρ

LEGEND

FROM DATA CALCN.
SHEETS DATED
20 I 62

TEST LOOP PERFORMANCE CURVES
12" ROTOR-& ELECTRODYNAMIC PUMPS
LIQUID METAL: NaK
PUMP #1796: PUMP ROTOR SPEED AS INDICATED
PUMP #1799: PUMP ROTOR SPEED ~1250 R.P.M.

FLOW RATE, NaK   G.P.M.

EFFECTIVE PRESSURE DEVELOPED, P.S.I.

Jmd
12 IX 61

LM ELECTRODYNAMIC PUMP MODEL P12H3-250B7.5 (Cb-1%Zr Cell, 0.060" wall)

PERFORMANCE CURVES. EXPERIMENTAL (I) AND THEORETICAL (II).

NaK, 400 °F. Pump Rotor Speed 1240 RPM. Field Current 3 amps (B=5150 gauss)

CAPACITY, GPM
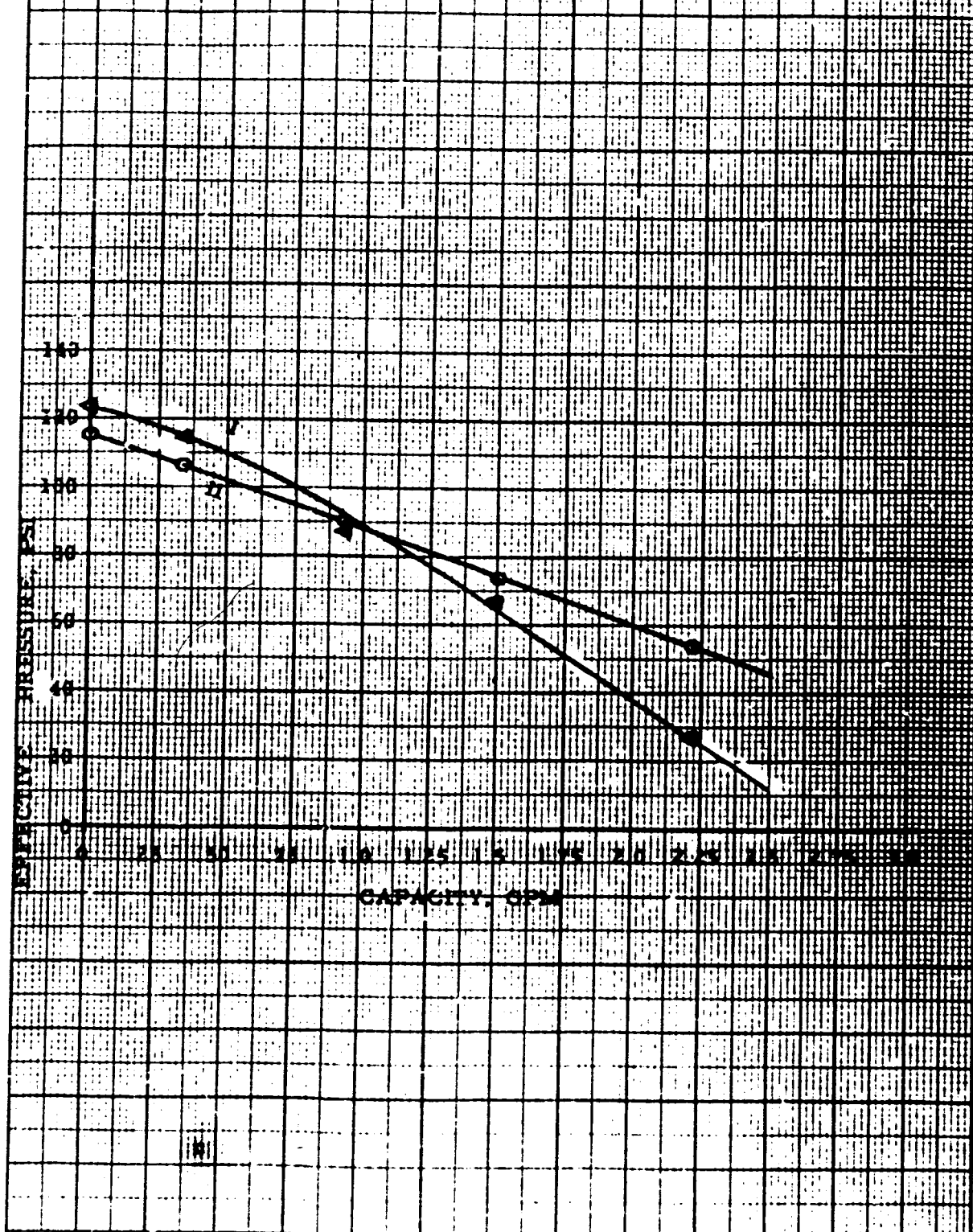
THEORETICAL PERFORMANCE CURVE PUMP 17006.
CALCULATED FOR Rb @ 1200 °F, B = 9250 GAUSS,
ROTOR RPM = 1453

EFFECTIVE PRESSURE DEVELOPED, PSI.

FLOW RATE RUBIDIUM, G.P.M.
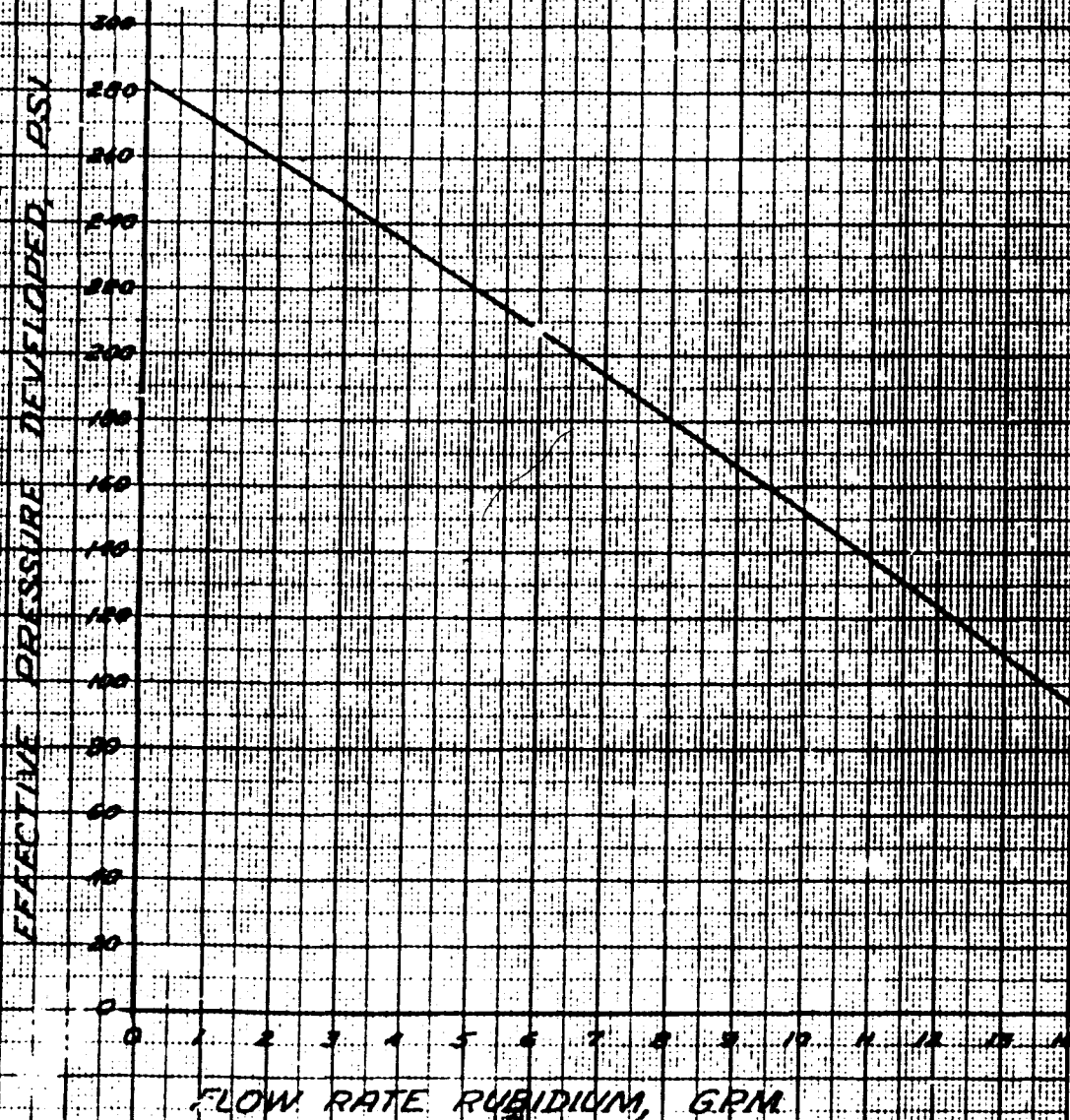
APPENDIX   IV

Miscellaneous Columnar and Graphical Data Useful for Pump

and

System Analysis Studies.

TABLE I

AREA (In.^{2}) REQUIRED ~~FOR VARIOUS FLOW CAPACITIES~~ IN GPM AND VARIOUS FLOW VELOCITIES IN FT./SECOND

Capacity GPM	Capacity In.^{2} Ft/sec.	Area (in.^{2}) for Flow Velocity of						
		V=1 ft/sec.	V=5 .ft/sec.	V=10 ft/sec.	V=20 ft/sec.	V=30 ft/sec.	V=40 ft/sec.	V=50 ft/sec.
1	.321	.321	.0642	.0321	.01605	.0107	.008025	.00642
5	1.605	1.605	.321	.1605	.08025	.0535	.0401	.0321
10	3.21	3.21	.642	.321	.1605	.107	.0802	.0642
15	4.815	4.815	.962	.4815	.24075	.1604	.1203	.0962
20	6.42	6.42	1.283	.642	..321	.214	.1605	.1283
25	8.02	8.02	1.605	.802	.401	.2673	.2005	.1605
30	9.64	9.64	1.925	.964	.482	.3213	.241	.1925
35	11.22	11.22	2.23	1.122	.561	.340	.280	.224
40	12.83	12.83	2.566	1.283	.6415	.4276	.3207	.256
45	14.45	14.45	2.89	1.445	.7225	.4816	.3612	.289
50	16.05	16.05	3.21	1.605	.8025	.5350	.4012	.321
55	17.65	17.65	3.53	1.765	.8825	.5883	.4412	.353
60	19.25	19.25	3.85	1.925	.9625	.6416	.4812	.385
65	20.85	20.85	4.17	2.085	1.0425	.695	.5212	.417
70	22.45	22.45	4.49	2.245	1.1225	.7483	.5612	.449
75	24.06	24.06	4.81	2.406	1.204	.8026	.6015	.481
80	25.68	25.68	5.14	2.568	1.284	.856	.642	.514
85	27.30	27.30	5.46	2.730	1.365	.910	.6825	.546
90	28.90	28.90	5.78	2.890	1.445	.9633	.7226	.578
95	30.50	30.50	6.1	3.050	1.525	1.0166	.7625	.61
100	32.1	32.1	6.42	3.21	1.605	1.070	.8025	.642

TABLE I (continued)

Area (In^2) Required for Various Flow Capacities in GPM & Various Flow Velocities in Ft/Second

Capacity GPM	Capacity In^2 Ft/sec.	Area (in. ^{2}) for Flow Velocity of						
		V=1 ft/sec.	V=5 ft/sec.	V=10 ft/sec.	V=20 ft/sec.	V=30 ft/sec.	V=40 ft/sec.	V=50 ft/sec.
125	40. 1	40. 1	. 8. 02	4. 01	2. 005	1. 3366	1. 0025	. 802
150	48. 2	48. 2	9. 64	4. 82	2. 41	1. 6066	1. 2050	. 964
175	56. 2	56. 2	11. 25	5. 62	2. 81	1. 8733	1. 405	1. 125
200	64. 2	64. 2	12. 84	6. 42	3. 21	2. 14	1. 605	1. 284
225	72. 2	72. 2	14. 44	7. 22	3. 61	2. 4066	1. 805	1. 445
250	80. 2	80. 2	16. 04	8. 02	4. 01	2. 673	2. 005	1. 605
275	88. 2	88. 2	17. 64	8. 82	4. 41	2. 94	2. 205	1. 764
300	96. 3	96. 3	19. 26	9. 63	4. 815	3. 210	2. 407	1. 926
350	112. 2	112. 2	22. 44	11. 22	5. 600	3. 740	2. 805	2. 244
400	128. 4	128. 4	25. 68	12. 84	6. 420	4. 280	3. 210	2. 568
450	144. 5	144. 5	28. 90	14. 45	7. 225	4. 816	3. 612	2. 89
500	160. 4	160. 4	32. 08	16. 04	8. 020	5. 346	4. 010	3. 21
550	176. 5	176. 5	35. 30	17. 65	8. 825	5. 883	4. 412	3. 53
600	192. 5	192. 5	38. 50	19. 25	9. 625	6. 416	4. 812	3. 85
650	208. 5	208. 5	41. 70	20. 85	10. 425	6. 950	5. 212	4. 17
700	224. 6	224. 6	44. 92	22. 46	11. 23	7. 486	5. 615	4. 49
750	240. 6	240. 6	48. 12	24. 06	12. 03	8. 020	6. 015	4. 81
800	256. 6	256. 6	51. 32	25. 66	12. 83	8. 553	6. 415	5. 13
850	272. 7	272. 7	54. 60	27. 27	13. 65	9. 100	6. 825	5. 46
900	288. 8	288. 8	57. 76	28. 88	14. 44	9. 626	7. 220	5. 776
950	305. 0	305. 0	61. 0	30. 5	15. 25	10. 166	7. 625	6. 10
1000	321. 0	321. 0	64. 2	32. 1	16. 050	10. 700	8. 025	6. 42

TABLE   II

AREA OF STANDARD TUBE SIZES for DIFFERENT

ELLIPSE   RATIOS

Tubing Size	Wall	I.D.	I.D. Area	Ellipse Area 2 - 1	Ellipse Area 3 - 1	Ellipse Area 4 - 1
3/8 - 20g	.035	.305	.07304	.061302	.04875	.03921
1/2 - 20g	.035	.430	.14522	.12188	.09691	.07794
5/8 - 20g	.035	.555	.24158	.20275	.16123	.12965
3/4 - 20g	.035	.680	.36317	.30480	.2423	.1949
1 - 18g	.049	.902	.6390	.5363	.42646	.34293
1-1/4 - 18g	.049	1.152	1.042	.8746	.69543	.55922
1-1/2 - 18g	.049	1.402	1.5437	1.2956	1.03026	.82847
2 - 16g	.065	1.870	2.7463	2.30496	1.8328	1.4738
2-1/2 - 16g	.065	2.370	4.4108	3.7019	2.9437	2.36718
3 - 14g	.083	2.834	6.3077	5.2940	4.2097	3.3852
4 - 14g	.083	3.834	11.545	9.6897	7.7051	6.196

TABLE OF TUBE SIZE AND CORRESPONDING ELLIPSE RADII

Tube	Wall	Tube Radius		2 - 1 Ellipse Radii	2 - 1 Ellipse O. D.	3 - 1 Ellipse Radii	3 - 1 Ellipse O. D.	4 - 1 Ellipse Radii	4 - 1 Ellipse O. D.
3/8 - 20g	.035	.1525	b	.0987	.2674	.0716	.2132	.0558	.1816
			a	.1974	.4648	.2148	.4996	.2232	.5164
1/2 - 20g	.035	.215	b	.1392	.3484	.1010	.2720	.0787	.2274
			a	.2784	.6268	.3030	.6760	.3148	.6996
5/8 - 20g	.035	.2775	b	.1797	.4294	.1303	.3306	.1016	.2732
			a	.3594	.7888	.3909	.8518	.4064	.8828
3/4 - 20g	.035	.340	b	.2202	.5104	.1597	.3894	.1245	.3190
			a	.4404	.9508	.4791	1.0282	.4980	1.0660
1 - 18g	.049	.451	b	.2921	.6802	.2119	.5218	.1652	.4284
			a	.5842	1.2665	.6357	1.3694	.6608	1.4196
1-1/4 - 18g	.049	.576	b	.3731	.8442	.2706	.6392	.2109	.5198
			a	.7462	1.5904	.8118	1.7216	.8436	1.7852
1-1/2 - 18g	.049	.701	b	.4541	1.0062	.3293	.7566	.2567	.6114
			a	.9082	1.9144	.9879	2.0738	1.0268	2.1516
2 - 16g	.065	.935	b	.5693	1.2686	.4393	1.0086	.3424	.8148
			a	1.1386	2.4072	1.3179	2.7658	1.3696	2.8692
2-1/2 - 16g	.065	1.185	b	.7676	1.6652	.5568	1.2436	.4340	.9980
			a	1.5352	3.2004	1.6704	3.4708	1.7360	3.6020
3 - 14g	.083	1.417	b	.9179	2.0018	.6658	1.4976	.5190	1.2040
			a	1.8358	3.8376	1.9974	4.1608	2.0760	4.3180
4 - 14g	.083	1.917	b	1.2418	2.6496	.9007	1.9674	.7021	1.5702
			a	2.4836	5.1332	2.7021	5.5702	2.8084	5.7828

TABLE III

TABLE IV

VELOCITY HEAD (IN FT. OF LIQUID AND IN LBS/IN^2) OF LIQUID SODIUM AT 1400° F

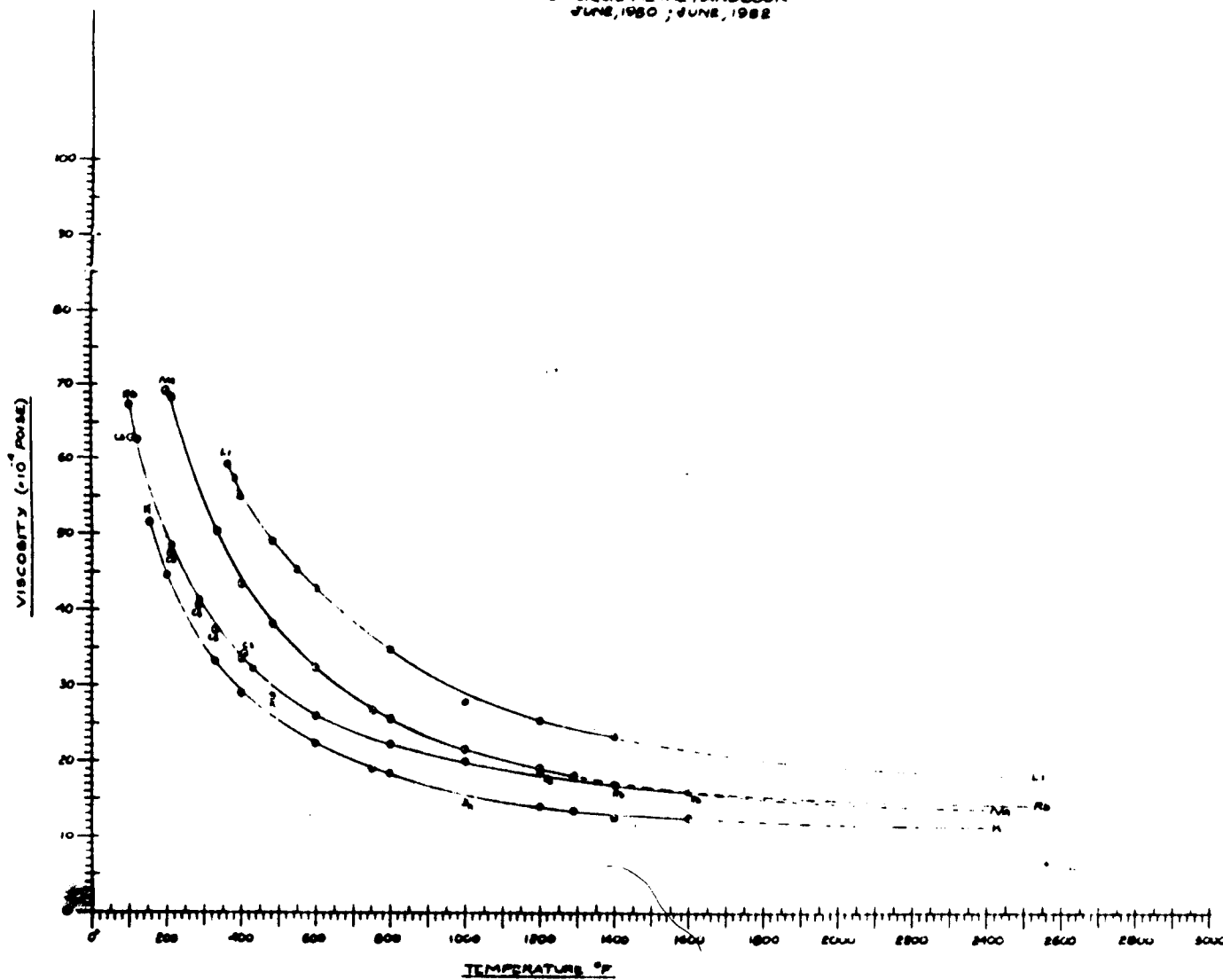| Velocity ft/sec.	$\dfrac{v^2}{2G}$ ft of liquid	$\dfrac{\text{ft} \times 47.9 \text{ #/ft.}^3}{144 \text{ in}^2/\text{ft}^2}$ lbs/ in^2
1...................	.01562	.00519
2...................	.0625	.02078
3...................	.1406	.04676
4...................	.2500	.08315
5...................	.3906	.1299
6...................	.5625	.1870
7...................	.7656	.2546
8...................	1.0000	.3326
9...................	1.2656	.4209
10...................	1.5625	.5196
11...................	1.8906	.6288
12...................	2.250	.7483
13...................	2.6406	.8782
14...................	3.0625	1.0186
15...................	3.515	1.1690
16...................	4.000	1.3304
17...................	4.515	1.5016
18...................	5.062	1.6836
19...................	5.640	1.875
20...................	6.250	2.0787
21...................	6.890	2.2916
22...................	7.562	2.5151
23...................	8.265	2.7489
24...................	9.000	2.9934
25...................	9.765	3.2478

TABLE IV (continued)

Velocity Head (in ft. of Liquid and in Lbs/in.^{2}) of Liquid Sodium at 1400° F

Velocity	$\dfrac{V^2}{2G}$ ft. of liquid	ft x $\dfrac{47.9 \text{ #/ft}^3}{144 \text{ in}^2/\text{ft}^2}$ lbs/ in^2
26.................	10.562	.... 3.5129
27.................	11.390	.... 3.7883
28.................	12.250	.... 4.0743
29.................	13.140	.... 4.3703
30.................	14.062	.... 4.6770
31.................	15.015	.... 4.9939
32.................	16.000	.... 5.3216
33.................	17.015	.... 5.6591
34.................	18.062	.... 6.0074
35.................	19.140	.... 6.3659
36.................	20.250	.... 6.7351
37.................	21.390	.... 7.1143
38.................	22.562	.... 7.5041
39.................	23.765	.... 7.9042
40.................	25.000	.... 8.3150
41.................	26.265	.... 8.7357
42.................	27.562	.... 9.1671
43.................	28.890	.... 9.6088
44.................	30.250	.... 10.0611
45.................	31.640	.... 10.523
46.................	33.062	.... 10.996
47.................	34.515	.... 11.479
48.................	36.000	.... 11.973
49.................	37.516	.... 12.4778
50.................	39.062	.... 12.9920

DENSITY VS. TEMPERATURE

Li, Na, K, Rb, Cs

Ref: Liquid Metal Handbook
AEC DEPT. OF THE NAVY
JUNE 1, 1950

DENSITY (Lbs/ft³) (vertical axis)

TEMPERATURE °F (horizontal axis)

	C	11405

LIQUID METALS, INC.
WESTFORD, MASS.

DENSITY VS TEMP
Li, Na, K, Rb, Cs

11405

VISCOSITY VS TEMPERATURE
Li, Na, K, Rb, Cs
Ref: LIQUID METAL HANDBOOK
JUNE, 1950 ; JUNE, 1952

VISCOSITY ($\times 10^4$ POISE)

TEMPERATURE °F

C 11409

LIQUID METALS, INC.

VISCOSITY VS TEMP
Li - Na - K - Rb - Cs

11409

VELOCITY HEAD (FT) VS LIQUID VELOCITY (FT/SEC)
STAGNATION PRESSURE (LBS/IN²) VS LIQUID VELOCITY (FT/SEC)

(a) FOR LIQ. NA @ 1000°F & VELOCITY OF 20 FT/SEC.
$$V^2/2g = 6.25 \ FT.$$
$$DENSITY = 47.9 \ LBS/FT^3$$

$$P = \frac{6.25 \ FT. \times 47.9 \ LBS/FT^3}{144 \ IN²/FT²} = 2.08 \ LBS/IN²$$

NOTE: PRESSURE CURVE IS DRAWN THROUGH ABOVE PT. PARALLEL
TO VELOCITY HEAD CURVE (V²/2g)

TEMPERATURE VS VAPOR PRESSURE OF
Li, Na, K, Rb, Cs, Hg, Hg

REF. DATA:
U.S. DEPT. OF THE INTERIOR
BUREAU OF MINES
BULLETIN 383

C 11307 B

LIQUID METALS, INC.
HARTFORD, MASS.

11307 B

HEAT CONTENT (BTU/LB) VS TEMPERATURE
Li, Na, K, Rb, CS, Mg, Hg
REF - BUREAU OF MINES, BULLETIN 584

TURE

LATENT HEAT OF VAPORIZATION (ΔH)

ELEMENT	TEMP	H(L)	H(g)	ΔH
Li	2500°F	2580 BTU/LB	4000 BTU/LB	1430 BTU/LB
NA	1675°F	536 BTU/LB	2366 BTU/LB	1830 BTU/LB
K	1430°F	275 BTU/LB	1150 BTU/LB	875 BTU/LB
CS	1275°F	75 BTU	300 BTU/LB	225 BTU/LB
RD	1265°F	120 BTU/LB	500 BTU/LB	380 BTU/LB
MG	2025°F	740 BTU/LB	2840 BTU/LB	2100 BTU/LB
HG	675°F	15 BTU/LB	147 BTU/LB	132 BTU/LB

Li

VAPORIZATION

Na

HEAT CONTENT (BTU/LB)

3000

2800

2600

2400

2200

2000

1800

1600

1400

K

1200

1000

800

Rb

600

Cs

400

Hg

200

0

°F
°R
°C
°K

D 1404A

ELECTRICAL RESISTIVITY Vs. TEMPERATURE
FOR LIQUID METALS
Na, K, Li, Rb, Cs, Hg
Bi, Pb, Sn, Zn

REF:
LIQUID METAL HANDBOOK
2ND EDT. JUNE, 1952

C 11407

LIQUID METALS, INC.
WESTFORD, MASS.

GRAPH ELEC. RES. VS TEMP.

11407